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13.) $F(x) = \ln|1+x| + C$

19.) $F(x) = -\frac{e^{-3x}}{3} + C$

31.) $F(x) = \frac{\tan(2x)}{2} + C$

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4.) a.) Note first that

$$f(0) = e^{-0}0 = 0$$

Additionally,

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} xe^{-x}$$
$$= \lim_{x\to\infty} \frac{x}{e^x}$$
$$= \frac{\infty}{\infty}$$

which is an indeterminate form. So we use L'Hospital's rule:

$$= \lim_{x\to\infty} \frac{1}{e^x}$$
$$= 0$$

Finally, since e^{-x} is always greater than zero, if $x > 0$ then $f(x) = xe^{-x} > 0$.

b.) Since $f(x) = xe^{-x}$,

$$f'(x) = x\frac{d}{dx}e^{-x} + e^{-x}\frac{d}{dx}x$$
$$= -xe^{-x} + e^{-x}$$
$$= e^{-x}(-x+1)$$

We wish to find where

$$f'(x) = e^{-x}(-x+1) = 0$$

Since e^{-x} is never zero, this occurs when

$$x = 1$$

So $(1, e^{-1})$ is a *local max* (we can check this by taking the 2nd derivative at $x = 1$, finding that $f''(1) < 0$). Note that it is greater than the endpoint at zero or the limit as $x \to \infty$. So $(1, e^{-1})$ is also a *global max*.

Moreover, the point $x = 0$, $y = 0$ is less than any other point on the curve (we showed in part (a) that $f(x) > 0$ everywhere else). So $(0, 0)$ is a *global min*.

c.) To find inflection points, we set the second derivative equal to zero:

$$\begin{aligned} f''(x) &= \tfrac{d}{dx}(-xe^{-x}) + \tfrac{d}{dx}(e^{-x}) \\ &= -x\tfrac{d}{dx}e^{-x} - (\tfrac{d}{dx}x)e^{-x} + \tfrac{d}{dx}e^{-x} \\ &= xe^{-x} - e^{-x} - e^{-x} \\ &= xe^{-x} - 2e^{-x} \\ &= (x-2)e^{-x} = 0 \end{aligned}$$

Since e^{-x} is never zero, this occurs when

$$x = 2$$

It is clear that the sign of f'' changes from negative to positive at $x = 2$. So $(2, 2e^{-2})$ is an inflection point.

d.) See graph page

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69.) If the endpoints are the same, the area under the curve is zero. So

$$\int_2^2 \cos(3x^2) = 0$$

74.) Geometrically, $\int_0^2 x^2 dx$ is the area under the curve from $x = 0$ to $x = 2$. Meanwhile, $\int_0^1 x^2 dx$ is the area under the curve from $x = 0$ to $x = 2$. Subtracting the latter from the former leaves the area from $x = 1$ to $x = 2$, which is expressed as $\int_1^2 x^2 dx$. This shows that

$$\int_1^2 x^2 dx = \int_0^2 x^2 dx - \int_0^1 x^2 dx$$

Additionally, we know that $\int_a^b = -\int_b^a$. So

$$-\int_0^1 x^2 dx = \int_1^0 x^2 dx$$

and therefore

$$\int_1^2 x^2 dx = \int_0^2 x^2 dx + \int_1^0 x^2 dx$$

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4.) If

$y = \int_0^x (1+u^4)du$

Let us call $f(u) = 1 + u^4$. Then, by the fundamental theorem of calculus (since $f(u)$ is continuous and differentiable everywhere),

$\frac{d}{dx} y = f(x) = 1 + x^4$

15.) Recall that Leibniz Rule states that

$\frac{d}{dx} \int_{g(x)}^{h(x)} f(t)dt = f[h(x)]h'(x) - f[g(x)]g'(x)$

In our case,

$f(t) = 1 + t$
$g(x) = 0$
$h(x) = 3x$

Plugging into Leibniz Rule,

$\frac{d}{dx} \int_0^{3x} (1+t)dt = f[3x]3 - f[0] \cdot 0$
$= (1+3x)3$

16.) In this case, we again use Leibniz Rule with

$f(t) = t^2 - 1$
$g(x) = 0$
$h(x) = 2x - 1$

So

$\int_0^{2x} (t^2 - 1)dt = f[2x-1]2 - f[0]0$
$= ((2x-1)^2 - 1)2$

45.) $\int \frac{2x^2 - x}{\sqrt{x}} dx = \int \frac{2x^2}{\sqrt{x}} - \frac{x}{\sqrt{x}} dx$
$= \int \frac{2x^2}{\sqrt{x}} dx - \int \frac{x}{\sqrt{x}} dx$
$= \int 2x^{\frac{3}{2}} dx - \int \sqrt{x} dx$
$= \frac{2}{5} \cdot 2x^{\frac{5}{2}} - \frac{2}{3} x^{\frac{3}{2}} + C$

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