Calc I

TA: Joe Citrone $\qquad$ Hw #3 Solutions

2.2.4

1. p. 101 #76:

$$\lim_{n\to\infty}\left(\frac{3n^2-5}{n^2}\right)=\lim_{n\to\infty}\frac{3-5/n^2}{1}=\lim_{n\to\infty}3-\cancel{5/n^2}^{\,0}=\boxed{3}$$

2. p. 101 #81

$$\lim_{n\to\infty}\frac{n+2^{-n}}{n}=\lim_{n\to\infty}\frac{n+\frac{1}{2^n}}{n}=\lim_{n\to\infty}1+\frac{1}{n2^n}=\lim_{n\to\infty}1+\cancel{\frac{1}{\infty}}^{\,0}=\boxed{1}$$

3. p. 101 #99

$$a_{n+1}=\frac{2}{a_n+2}$$

Find Fixed Pts: $L=\frac{2}{L+2}\Rightarrow L(L+2)=2\Rightarrow L^2+2L-2=0$

use quadratic equation: $L=\frac{-2\pm\sqrt{4+8}}{2}=\frac{-2\pm\sqrt{12}}{2}=\frac{-2\pm2\sqrt{3}}{2}=\boxed{-1\pm\sqrt{3}}$

3.1.3

p. 128 #41

$$\lim_{x\to3}\left(2x^2-\frac{1}{x}\right)=2(3)^2-\frac{1}{3}=\boxed{17\tfrac{2}{3}=\frac{53}{3}}$$

p. 128 #42

$$\lim_{x\to-2}\left(\frac{x^2}{2}-\frac{2}{x^2}\right)=\frac{4}{2}-\frac{2}{4}=\frac{8-2}{4}=\frac{6}{4}=\boxed{\frac{3}{2}=1\tfrac{1}{2}}$$

p. 128 #44

$$\lim_{x\to1}\frac{x^3-1}{x+2}=\frac{1-1}{3}=\boxed{0}$$

3.2.3

p. 137 #5 $\qquad$ * iff = if and only if

$f(x)$ is continuous at $x=2$ iff $f(2)=\lim_{x\to2}f(x)$

but $f(2)=3$ and $\lim_{x\to2}f(x)=\lim_{x\to2}\frac{x^2-x-2}{x-2}=\lim_{x\to2}\frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}}=3$

$\therefore f(x)$ is continuous

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p. 137 #8

$$f(x) = \begin{cases} \frac{x^2+x-2}{x-1} & \text{if } x \neq 1 \\ a & \text{if } x = 1 \end{cases}$$

To make $f(x)$ continuous at $x=1$, $f(1)$ must equal $\lim_{x\to 1} f(x)$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2+x-2}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}} = 3$$

$$\therefore \boxed{a = 3}$$

p. 137 #10

$$f(\pm 1) = 0 \quad \text{but} \quad \lim_{x\to \pm 1} f(x) = \lim_{x\to \pm 1} \frac{1}{x^2-1} = \frac{1}{1-1} = \frac{1}{0} = \pm\infty \therefore \text{DNE}$$

$\therefore$ discontinuous

p. 138 #46

$$\lim_{x\to 0} \frac{5-\sqrt{25+x^2}}{2x^2} = \lim_{x\to 0} \frac{5-\sqrt{25+x^2}}{2x^2} \cdot \left(\frac{5+\sqrt{25+x^2}}{5+\sqrt{25+x^2}}\right) = \lim_{x\to 0} \frac{25-25+x^2}{2x^2(5+\sqrt{25+x^2})}$$

$$= \lim_{x\to 0} \frac{-1}{2(5+\sqrt{25+x^2})} = \frac{-1}{2(5+5)} = \boxed{\frac{-1}{20}}$$