Hw 5 Soln's

p.152 2. a) Graph

b) $f(-3) = NEG,\ f(-1) = Pos \therefore$ by IVT

3. a) Graph

b) $+, - \therefore$ by IVT

4. a) Graph

b) $+, - \therefore$ by IVT

13. $x^2 - 4 = (x-2)(x+2) \therefore$ roots: ± 2

14. $\lim_{x \to \infty} p(x) = \infty \quad \lim_{x \to -\infty} p(x) = -\infty \therefore$ by IVT $\exists c$ s.t. $p(c) = 0$

p. 177. 13. b) slope of tan $= -10 \quad \therefore y = -10x - 5$

c) graph

14. b) slope of tan $= -4 \quad \therefore y = -4x + 2$

c) graph

16. a) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h} = \dfrac{x - x - h}{h(x+h)x} = \dfrac{-1}{x^2}$

at $x = 2,\ f'(x) = -\frac{1}{4}$

b) Eq of normal line at $(2, \frac{1}{2})$ is $y - \frac{1}{2} = \left(\frac{-1}{4}\right)(x-2) = \frac{x}{4} + 1$

c) Graph

26. $y = \frac{x}{4} + \frac{3}{4}$

53. a) $s\left(\frac{3}{4}\right) = 30 \quad s(1) = \frac{160}{3}$ b) $\frac{280}{3}$ c) 80 ; 80

Calc I Soln's - Hw #5

p.152 #2) $f(x) = x^3 - 2x + 3 \quad -3 \le x \le -1$

b) $f(-3) = -27 + 6 + 3 =$ NEGATIVE

$f(-1) = -1 + 2 + 3 =$ POSITIVE

$\therefore$ By IVT, $\exists c \in [-3,-1)$ s.t. $f(c) = 0$

3) $f(x) = \sqrt{x^2+2} \quad 1 \le x \le 2$

b) Define $g(x) = \sqrt{x^2+2} - 2$ which is clearly continuous on $[1,2]$

$g(1) = \sqrt{3} - 2 =$ NEGATIVE

$g(2) = \sqrt{6} - 2 =$ POSITIVE

$\therefore$ By IVT, $\exists c \in (1,2)$ s.t. $g(c) = 0$

ie $\sqrt{x^2+2} - 2 = 0$

ie $\sqrt{x^2+2} = 2$

4. b) $f(-1) =$ NEG

$f(1) =$ Pos

$\therefore$ By IVT, $\exists\, c \in (-1,1)$ s.t. $f(c) = 0$

ie $\sin x = x$ has a solution in $(-1,1)$

13. $y = x^2 - 4 = (x-2)(x+2)$

roots are then ± 2

Also, by IVT, can show that graph crosses real line exactly twice

14. for a polynomial $p(x)$ of odd degree,

$\lim_{x\to\infty} p(x) = \infty, \quad \lim_{x\to-\infty} p(x) = -\infty \quad \therefore$ by IVT, $\exists c \in \mathbb{R}$ s.t. $p(c) = 0$

p. 177 13 b) $y = 5x^2$

$(-1, 5)$ clearly on graph since $5 = 5(-1)^2$

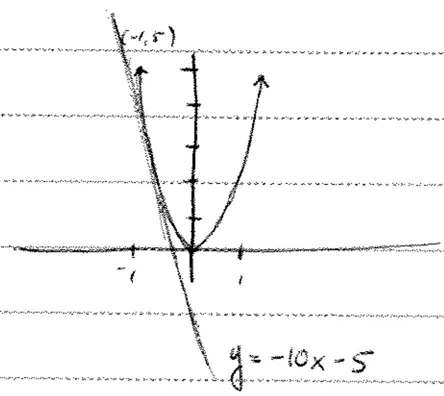
slope of tangent line: $10x$

at $x = -1$: -10

$y - 5 = -10(x + 1)$

$\boxed{y = -10x - 5}$

c)



14 b) $f(x) = -2x^2$

$f'(x) = -4x$

$\therefore$ slope of tan line at $(1, -2) = -4$

$y + 2 = -4(x - 1)$

$y = -4x + 2$

c) Graph

p. 177 #16 a) $y = \frac{1}{x} = f(x)$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{x - x - h}{h(x+h)x} = \frac{-1}{x^2}$

at $x = 2$, $\frac{-1}{4} = f'(2)$

b) $(2, \frac{1}{2})$ clearly on graph of $y = \frac{1}{x}$

Eq of normal line at $(2, \frac{1}{2})$ is $y - \frac{1}{2} = (-\frac{1}{4})(x - 2)$

$y = -\frac{x}{4} + \frac{1}{2} + \frac{1}{2} = \boxed{\frac{-x}{4} + 1}$

c) Graph it (you can do this...)

26. $f(x) = \sqrt{x-1}$ at $(5, 2)$

$f'(x) = \frac{1}{2}(x-1)^{-1/2}$

$f'(5) = \frac{1}{2}(4)^{-1/2} = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x - 5)$

$y = \frac{x}{4} + \frac{3}{4}$ tangent line

normal line has slope -4 $\therefore$ $y - 2 = -4(x - 5)$

$\boxed{y = -4x + 22}$

33 $s(t) = \frac{160}{3}t^2 \quad 0 \le t \le 1$

a) at $t = 3/4$, $s(3/4) = 30$ $\quad s(1) = \frac{160}{3}$

b) average velocity $= \frac{\Delta s}{\Delta t} = \frac{\frac{160}{3} - \frac{90}{3}}{1 - 3/4} = \frac{\frac{70}{3}}{1/4} = \frac{280}{3}$

c) $v(t) = s'(t) = \frac{320}{3}t \quad \therefore v(3/4) = 80$

speed $= |v(t)| = |v(3/4)| = 80$