Calc I

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Hw 8 solns

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2. $f(x) = \sqrt{x-1} \quad x \geq 1$

i) $f^{-1}(x) = x^2 + 1$

$\frac{\partial}{\partial x} f^{-1}(x) = 2x$

ii) $f'(x) = \frac{1}{2}(x-1)^{-1/2}$

$f'(f^{-1}(x)) = \frac{1}{2x}$

$\frac{1}{f'(f^{-1}(x))} = 2x = \frac{\partial}{\partial x} f^{-1}(x)$

5. $f(x) = 3 - 2x^3 \quad x \geq 0$

i) $f^{-1}(x) = \sqrt[3]{\frac{3-x}{2}}$

$\frac{\partial}{\partial x} f^{-1}(x) = \frac{1}{3}\left(\frac{3-x}{2}\right)^{-2/3}\left(-\frac{1}{2}\right) = -\frac{1}{6}\left(\frac{3-x}{2}\right)^{-2/3}$

ii) $f'(x) = -6x^2$

$\frac{1}{f'(f^{-1}(x))} = \frac{\partial}{\partial x} f^{-1}(x) = -\frac{1}{6}\left(\frac{3-x}{2}\right)^{-2/3}$

6. $f(x) = \frac{2x^2-1}{x^2-1} \quad x > 1$

i) $f^{-1}(x) = \sqrt{\frac{x-1}{x-2}} \quad x > 2$

$\frac{\partial}{\partial x} f^{-1}(x) = \frac{1}{2}\left(\frac{x-1}{x-2}\right)^{-1/2}\left(\frac{(x-2)-(x-1)}{(x-2)^2}\right) = \frac{1}{2}\left(\frac{x-1}{x-2}\right)^{-1/2}\left(\frac{-1}{(x-2)^2}\right)$

ii) $f'(x) = \frac{-2x^2}{(x^2-1)^2}$

$\frac{\partial}{\partial x} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{2}\left(\frac{x-1}{x-2}\right)^{-1/2}\left(\frac{-1}{(x-2)^2}\right)$

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10. $f(x) = \sqrt{5+x^2}$ $x \geq 0$ $\quad f'(x) = \frac{1}{2}(5+x^2)^{-1/2}(2x)$ $\quad f'(2) = \frac{2}{3}$

$\frac{d}{dx} f^{-1}(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(2)} = \frac{1}{2/3} = \frac{3}{2}$

19. $f(x) = x^5 + x + 1$ $\quad x \in [-1,1]$ $\quad f'(x) = 5x^4 + 1$

$\frac{d}{dx} f^{-1}(1) = \frac{1}{f'(0)} = \frac{1}{1} = 1$

21. $f(x) = e^{-x^2/2} + 2x$ $\quad f'(x) = e^{-\frac{x^2}{2}}(x) + 2$

$\frac{d}{dx} f^{-1}(1) = \frac{1}{f'(0)} = \frac{1}{2}$

30. $f(x) = \ln(x^3 - 1)$

$f'(x) = \frac{1}{x^3-1} \cdot 3x^2$

37. $f(x) = \ln \frac{x}{x+1} = \ln x - \ln(x+1)$

$f'(x) = \frac{1}{x} - \frac{1}{x+1} = \frac{1}{x(x+1)}$

45. $f(x) = \ln(\tan x^2)$

$f'(x) = \frac{1}{\tan x^2} \cdot \sec^2 x^2 \cdot 2x$

67. $f(x) = x^{\ln x}$

$\ln y = \ln x \cdot \ln x = (\ln x)^2$

$\frac{y'}{y} = \frac{2 \ln x}{x}$

$y' = \frac{2 \ln x}{x} \cdot y = \frac{2 \ln x}{x} \cdot x^{\ln x}$

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76. $y = \dfrac{e^{x-1}\sin^2 x}{(x^2+5)^{2x}}$

$$\ln y = x - 1 + \ln \sin^2 x - 2x \ln (x^2+5)$$

$$\frac{y'}{y} = 1 + 2\frac{\cos x}{\sin x} - \left[\frac{4x^2}{x^2+5} + 2\ln(x^2+5)\right]$$

$$= 1 + 2\cot x - \frac{4x^2}{x^2+5} - 2\ln(x^2+5)$$

$$y' = \left[1 + 2\cot x - \frac{4x^2}{x^2+5} - 2\ln(x^2+5)\right] \cdot \left(\frac{e^{x-1}\sin^2 x}{(x^2+5)^{2x}}\right)$$