Review Exercises

(1) Find the domain of the following functions

$f(x) = \sqrt{x^2 - x + 11}$, $f(x) = \sqrt[3]{x-3}$, $f(x) = \dfrac{x+3}{\sqrt{x^2-1}}$

$f(x) = \dfrac{x}{\ln(x)}$, $f(x) = \sin^{-1}(x)$, $f(x) = \dfrac{x+1}{x^2-2x-3}$

(2) Compute the following limits (if they exist)

$\lim_{x\to 1} \dfrac{|x-1|}{x}$, $\lim_{x\to 0} \dfrac{|x|}{2x}$, $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$

$\lim_{x\to 0+} \cos\left(\frac{1}{x}\right)$, $\lim_{x\to\infty} \dfrac{2}{3x-1}$, $\lim_{x\to 2} \dfrac{x+1}{x^2-x-1}$

$\lim_{x\to 0} \dfrac{\sin x}{\sqrt{x}}$, $\lim_{x\to 0+} \dfrac{\ln x}{x}$, $\lim_{x\to -\infty} \dfrac{x^2-7x+24}{-x^2+1}$

$\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$, $\lim_{x\to 0} \sin^{-1}(x)$, $\lim_{x\to 0} x^2 \cot g(2x)$

(3) Find the largest interval on which $f(x)$ is continuous:

$f(x) = \dfrac{x(x+1)(x+2)}{\sqrt{(x-1)(x-2)}}$, $f(x) = \begin{cases} \dfrac{\cos x - 1}{2x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, $f(x) = \begin{cases} \dfrac{2}{x} & x > 0 \\ 0 & x \leq 0 \end{cases}$

$f(x) = \dfrac{2+7x}{\sqrt{5x^2-1}}$, $f(x) = \begin{cases} \dfrac{\sin x}{\sqrt{x}} & x > 0 \\ x & x \leq 0 \end{cases}$

(4) Find the largest interval on which $f'(x)$ is continuous, for $f(x)$ as in (3)

(5) How many real roots do:

$x^3 + 2x + 1 = 0$ $\qquad$ $x^2 + 2x + 25 = 0$

$x^4 - 2x^2 + 2 = 0$ $\qquad$ $2x^3 - 5x + 7 = 0$

have? Locate them between consecutive integers.

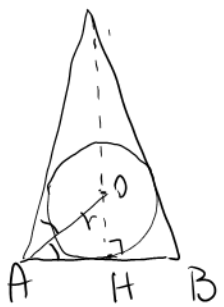
(6) Show that $f(x) = 3x^3 + 7x - 1$ has exactly one real root.

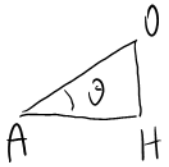
(7) Find local and absolute extrema for the following functions

$f(x) = 2x^2 + |x| + 1$ on $I = [-\frac{1}{2}, 1]$

$f(x) = 3\sqrt{x} - 2x\sqrt{x}$ on $Dom(f)$

(8) Find the dimensions of the isosceles triangle of least area that circumscribes a circle of radius r





HINT: Set

$\overline{AH} = r\,\mathrm{cotg}(\theta)$

$\overline{AB} = \ldots$

$\overline{AC} = \ldots$

Area $= A(\theta) = \ldots$

(9) Find concavity and inflection points of

$f(x) = x^3 - 3x^2 + 7x + 2$

(10) Sketch the graph of the following functions

$f(x) = \dfrac{x-2}{x^3}$, $f(x) = \cos(2x) + 2\cos(x)$, $f(x) = \dfrac{x^2-2}{x+2}$

That means find: Dom(f), asymptotes (horizontal, vertical, oblique) if any, compute local and absolute extrema, determine concavity, inflection points

(11) Compute the following integrals

$\int_0^1 x^6\sqrt{x^2+2}\,dx$, $\int \sin^2(x)\,dx$, $\int_0^{\pi} x\cos(x)\,dx$

$\int \dfrac{x^2+1}{x+2}\,dx$, $\int x^2 e^x\,dx$, $\int_2^3 x^{2-x}\,dx$

$\int \sin^{-1}(x)\,dx$, $\int_{-1}^{3} \dfrac{x^2+2x-3}{(x-2)(x^2+1)}\,dx$

(12) Compute the area of the region Ω between the graphs of $y = \sin x$ and $y = \cos x$ and the X-axis if $x \in [0, \pi/2]$