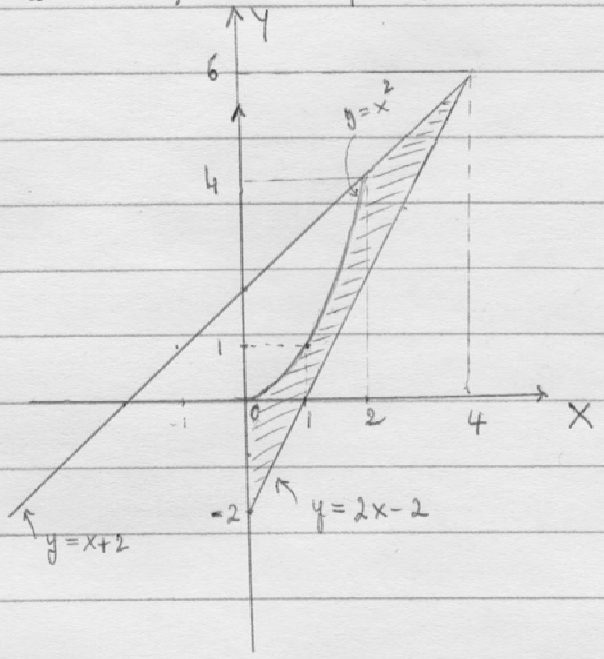
Consider the bounded region outside the graph of $f(x) = x^2$ and in between the graphs of $g(x) = x+2$, $h(x) = 2x-2$ and the y-axis.

Revolve it about X and Y axes: compute the volumes.

Sol



① Revolution about X-axis (cylindrical shell method)

$$vol = \int_{y=-2}^{y=0} 2\pi y\left(\frac{y}{2}+1\right)dy + \int_{y=0}^{y=4} 2\pi y\left[\frac{y}{2}+1-\sqrt{y}\right]dy + \int_{y=4}^{y=6} 2\pi y\left[\frac{y}{2}+1-y+2\right]dy$$

② Revolution about Y-axis (cylindrical shell method)

vol ∄ (this is because the hypotheses of the cylindrical shell method must apply)

note: for the revolution about the y-axis we consider the region in the 1st quadrant (ie. with $x \geq 0$ and $y \geq 0$)

$$vol = \int_{x=0}^{x=1} 2\pi x \cdot x^2\,dx + \int_{x=1}^{x=2} 2\pi x\left[x^2-2x+2\right]dx + \int_{x=2}^{x=4} 2\pi x\left[x+2-2x+2\right]dx$$