HW # 1 Solutions

Ch 4 p 244

12] $\int_0^1 (\sqrt[4]{u} + 1)^2 \, du$ Put $u = t^4 \Rightarrow du = 4t^3 dt$

$$\int_0^1 (\sqrt[4]{u} + 1)^2 du = \int_0^1 (t+1)^2 4t^3 dt$$

Expand and integrate the polynomial.

17] $$\int \frac{x+2}{\sqrt{x^2+4x}} dx = \frac{1}{2} \int \frac{2x+4}{\sqrt{x^2+4x}} dx$$

$$= (x^2+4x)^{1/2}$$

20] $\int \sin x \cos(\cos x) \, dx$.

Put $\cos x = u \Rightarrow -\sin x \, dx = du$

$$\therefore \int \sin x \cos(\cos x) dx = -\int \cos u \, du$$

$$= -\sin u$$

$$= -\sin(\cos x)$$

Part 6.1

11] $\int \arctan 4t \, dt$. Put $4t = \tan\theta$.

$\Rightarrow 4\,dt = \sec^2\theta \, d\theta$.

$$\Rightarrow \int \arctan 4t \, dt = \frac{1}{4}\int \theta \sec^2\theta \, d\theta$$

$$= \frac{1}{4}\int \theta \, d(\tan\theta) = \frac{1}{4}\int \left(d(\theta\tan\theta) - \tan\theta \, d\theta\right)$$

$$= \frac{1}{4}\int d(\theta\tan\theta) - \frac{1}{4}\int \tan\theta \, d\theta$$

$$= \frac{1}{4}\left(\theta\tan\theta + \ln\cos\theta\right)$$

13] Say $\int e^{2\theta}\sin 3\theta \, d\theta = I$.

Then, $I = \int e^{2\theta}\sin 3\theta \, d\theta = -\int \frac{1}{2} \sin 3\theta \, d(e^{2\theta})$

$$= -\frac{1}{2}\int \sin 3\theta \, d(e^{2\theta}) = -\frac{1}{2}\int d(\sin 3\theta \, e^{2\theta}) + \frac{1}{2}\int e^{2\theta} \, d(\sin 3\theta)$$

$$= -\frac{\sin 3\theta \; e^{2\theta}}{2} + \frac{3}{2}\int e^{2\theta}\cos 3\theta \, d\theta$$

$$= -\frac{\sin 3\theta \, e^{2\theta}}{2} + \frac{3}{2}\left(\frac{1}{2}\int \cos 3\theta \, d(e^{2\theta})\right)$$

$$= -\frac{e^{2\theta}\sin 3\theta}{2} + \frac{3}{4}\int \left(d(e^{2\theta}\cos 3\theta) - e^{2\theta} d(\cos 3\theta)\right)$$

$$= -\frac{e^{2\theta}\sin 3\theta}{2} + \frac{3}{4}e^{2\theta}\cos 3\theta + \frac{9}{4}\int e^{2\theta}\sin 3\theta \, d\theta$$

$$= -\frac{e^{2\theta}\sin 3\theta}{2} + \frac{3}{4}e^{2\theta}\cos 3\theta + \frac{9I}{4}$$

Now solve for I.

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$$\int_1^2 \frac{\ln x}{x^2}\,dx = \int_1^2 \ln x \, d\left(-\frac{1}{x}\right)$$

$$= \int_1^2 d\left(-\frac{\ln x}{x}\right) - \int_1^2 \left(-\frac{1}{x}\right) d(\ln x)$$

$$= -\frac{\ln x}{x}\Big|_1^2 + \int_1^2 \frac{dx}{x^2}$$

$$= -\ln 2 + \int_1^2 \frac{dx}{x^2}$$

6.2|

12 $\int x \cos^2 x \, dx$

$$= \frac{1}{2}\int x(\cos 2x + 1)\, dx = \frac{1}{2}\int x\cos 2x\, dx + \frac{1}{2}\int x\, dx$$

$$= \frac{1}{2}\int \frac{x}{2}\, d(\sin 2x) + \frac{x^2}{4}$$

$$= \frac{1}{4}\int \left(d(x\sin 2x) - \sin 2x\, dx\right) + \frac{x^2}{4}$$

$$= \frac{1}{4}x\sin 2x - \frac{1}{4}\int \sin 2x\, dx + \frac{x^2}{4}$$

$$= \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + \frac{x^2}{4}$$

16| $\int \cos^2 x \sin 2x\, dx = \int \frac{\cos 2x + 1}{2}\sin 2x\, dx$

$$= \frac{1}{2}\int \sin 2x \cos 2x\, dx + \frac{1}{2}\int \sin 2x\, dx$$

$$= \frac{1}{4}\int \sin 4x\, dx + \frac{1}{2}\int \sin 2x\, dx$$

$$= -\frac{1}{4}\frac{\cos 4x}{4} - \frac{1}{2}\frac{\cos 2x}{2}$$

~~52|~~ $\cancel{\int \frac{dx}{x^2\sqrt{16x^2-9}}}$ ~~Put~~ $\cancel{x^2 = t \Rightarrow 2x\,dx = dt}$

$\cancel{\Rightarrow dx = d}$

52| $\int \frac{dx}{x^2\sqrt{16x^2-9}}$ Put $x = \frac{3}{4}\sec\theta$.

$\Rightarrow dx = \frac{3}{4}\sec\theta \tan\theta \, d\theta$.

$$\int \frac{dx}{x^2\sqrt{16x^2-9}} = \int \frac{\frac{3}{4}\sec\theta \tan\theta \, d\theta}{\left(\frac{3}{4}\right)^2 \sec^2\theta \, \sqrt{9(\sec^2\theta - 1)}}$$

$$= \frac{4}{3}\,\frac{1}{3}\int \frac{\sec\theta \tan\theta \, d\theta}{\sec^2\theta \tan\theta}$$

$$= \frac{4}{9}\int \cos\theta \, d\theta.$$

59| $\int \frac{1}{\sqrt{9x^2+6x-8}}\,dx$

$$= \frac{1}{3}\int \frac{1}{\sqrt{x^2 + \frac{2x}{3} - \frac{8}{9}}}\,dx$$

$$= \frac{1}{3}\int \frac{1}{\sqrt{\left(x+\frac{1}{3}\right)^2 - 1}}\,dx$$