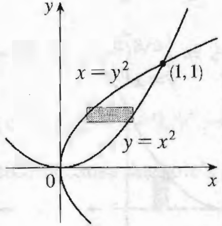
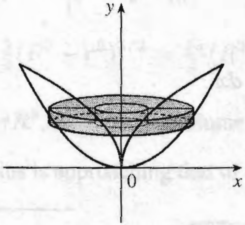
8. By slicing: $V = \int_0^1 \pi \left[(\sqrt{y})^2 - (y^2)^2 \right] dy = \pi \int_0^1 (y - y^4)\, dy = \pi \left[\frac{1}{2}y^2 - \frac{1}{5}y^5 \right]_0^1 = \pi\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3\pi}{10}$



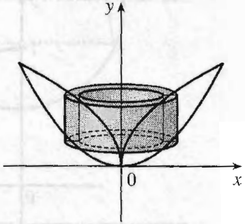


By cylindrical shells:

$$V = \int_0^1 2\pi x\left(\sqrt{x} - x^2\right) dx = 2\pi \int_0^1 (x^{3/2} - x^3)\, dx$$

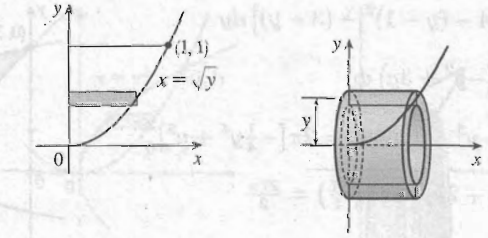
$$= 2\pi \left[\frac{2}{5}x^{5/2} - \frac{1}{4}x^4 \right]_0^1 = 2\pi\left(\frac{2}{5} - \frac{1}{4}\right)$$

$$= 2\pi\left(\frac{3}{20}\right) = \frac{3\pi}{10}$$



$V = \int_0^1 2\pi y \sqrt{y}\, dy = 2\pi \int_0^1 y^{3/2}\, dy$

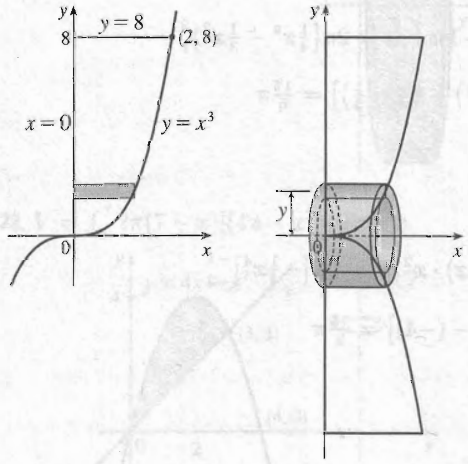
$= 2\pi \left[\frac{2}{5} y^{5/2}\right]_0^1 = \frac{4\pi}{5}$



$V = 2\pi \displaystyle\int_0^8 [y(\sqrt[3]{y} - 0)]\, dy$

$= 2\pi \displaystyle\int_0^8 y^{4/3}\, dy = 2\pi \left[\frac{3}{7} y^{7/3}\right]_0^8$

$= \dfrac{6\pi}{7}(8^{7/3}) = \dfrac{6\pi}{7}(2^7) = \dfrac{768\pi}{7}$

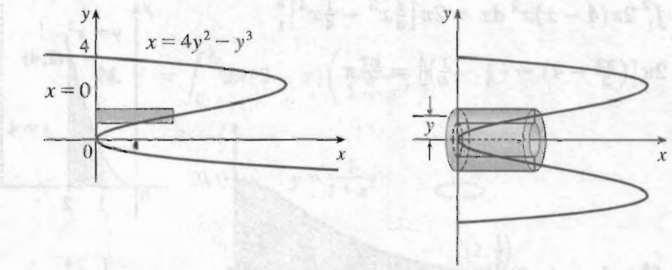


$V = 2\pi \displaystyle\int_0^4 [y(4y^2 - y^3)]\, dy$

$= 2\pi \displaystyle\int_0^4 (4y^3 - y^4)\, dy$

$= 2\pi \left[y^4 - \frac{1}{5} y^5\right]_0^4 = 2\pi\left(256 - \frac{1024}{5}\right)$

$= 2\pi\left(\frac{256}{5}\right) = \frac{512\pi}{5}$



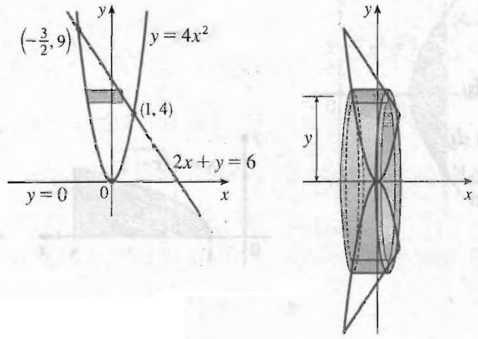
The curves intersect when $4x^2 = 6 - 2x \;\Leftrightarrow\; 2x^2 + x - 3 = 0 \;\Leftrightarrow\; (2x+3)(x-1) = 0 \;\Leftrightarrow\; x = -\frac{3}{2}$ or 1.

Solving the equations for x gives us $y = 4x^2 \;\Rightarrow\; x = \pm\frac{1}{2}\sqrt{y}$ and $2x + y = 6 \;\Rightarrow\; x = -\frac{1}{2}y + 3$.

$$V = 2\pi \int_0^4 \left\{y\left[\left(\tfrac{1}{2}\sqrt{y}\right) - \left(-\tfrac{1}{2}\sqrt{y}\right)\right]\right\} dy + 2\pi \int_4^9 \left\{y\left[\left(-\tfrac{1}{2}y + 3\right) - \left(-\tfrac{1}{2}\sqrt{y}\right)\right]\right\} dy$$

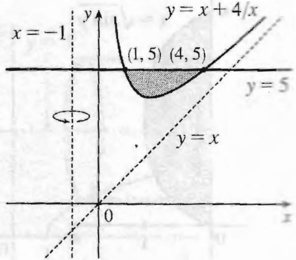
$$= 2\pi \int_0^4 \left(y\sqrt{y}\right) dy + 2\pi \int_4^9 \left(-\tfrac{1}{2}y^2 + 3y + \tfrac{1}{2}y^{3/2}\right) dy = 2\pi\left[\tfrac{2}{5}y^{5/2}\right]_0^4 + 2\pi\left[-\tfrac{1}{6}y^3 + \tfrac{3}{2}y^2 + \tfrac{1}{5}y^{5/2}\right]_4^9$$

$$= 2\pi\left(\tfrac{2}{5}\cdot 32\right) + 2\pi\left[\left(-\tfrac{243}{2} + \tfrac{243}{2} + \tfrac{243}{5}\right) - \left(-\tfrac{32}{3} + 24 + \tfrac{32}{5}\right)\right] = \tfrac{128}{5}\pi + 2\pi\left(\tfrac{433}{15}\right) = \tfrac{1250}{15}\pi = \tfrac{250}{3}\pi$$



$$= 2\pi\left[(-4+8-4) - \left(-\tfrac{1}{4}+1-1\right)\right] = \tfrac{\pi}{2}$$

35. Use shells:



$$V = \int_1^4 2\pi[x-(-1)][5-(x+4/x)]\,dx = 2\pi \int_1^4 (x+1)(5-x-4/x)\,dx$$

$$= 2\pi \int_1^4 (5x - x^2 - 4 + 5 - x - 4/x)\,dx$$

$$= 2\pi \int_1^4 (-x^2 + 4x + 1 - 4/x)\,dx = 2\pi\left[-\tfrac{1}{3}x^3 + 2x^2 + x - 4\ln x\right]_1^4$$

$$= 2\pi\left[\left(-\tfrac{64}{3} + 32 + 4 - 4\ln 4\right) - \left(-\tfrac{1}{3} + 2 + 1 - 0\right)\right]$$

$$= 2\pi(12 - 4\ln 4) = 8\pi(3 - \ln 4)$$

[illegible] Use disks: $V = \pi \int_0^2 \left[\sqrt{1-(y-1)^2}\right]^2 dy = \pi \int_0^2 \left(2y - y^2\right) dy = \pi\left[y^2 - \frac{1}{3}y^3\right]_0^2 = \pi\left(4 - \frac{8}{3}\right) = \frac{4}{3}\pi$

[illegible] Using shells, we have

4. $y^2 = 4(x+4)^3,\ \ y > 0 \ \Rightarrow \ y = 2(x+4)^{3/2} \ \Rightarrow \ dy/dx = 3(x+4)^{1/2} \Rightarrow$

$1 + (dy/dx)^2 = 1 + 9(x+4) = 9x + 37$. So

$$L = \int_0^2 \sqrt{9x+37}\,dx \quad \begin{bmatrix} u = 9x+37, \\ du = 9\,dx \end{bmatrix} \quad = \int_{37}^{55} u^{1/2}\left(\frac{1}{9}\,du\right) = \frac{1}{9}\cdot\frac{2}{3}\left[u^{3/2}\right]_{37}^{55} = \frac{2}{27}\left(55\sqrt{55} - 37\sqrt{37}\right)$$

$y = \ln x \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x} \Rightarrow \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2} = \sqrt{1 + \left(\dfrac{1}{x}\right)^2} = \dfrac{\sqrt{1+x^2}}{x}$. So $L = \displaystyle\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x}\,dx.$

Now let $v = \sqrt{1+x^2}$, so $v^2 = 1 + x^2$ and $v\,dv = x\,dx$. Thus

$$L = \int_{\sqrt{2}}^{2} \frac{v}{v^2-1}\,v\,dv = \int_{\sqrt{2}}^{2}\left(1 + \frac{1/2}{v-1} - \frac{1/2}{v+1}\right)dv = \Big[v + \tfrac{1}{2}\ln|v-1| - \tfrac{1}{2}\ln|v+1|\Big]_{\sqrt{2}}^{2}$$

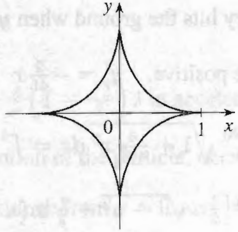
$$= \left[v - \tfrac{1}{2}\ln\left|\frac{v+1}{v-1}\right|\right]_{\sqrt{2}}^{2} = 2 - \tfrac{1}{2}\ln 3 - \sqrt{2} + \frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) = 2 - \sqrt{2} + \ln(\sqrt{2}+1) - \tfrac{1}{2}\ln 3$$

Use Formula 23 in the Table of Integrals.

25. $y^{2/3} = 1 - x^{2/3} \;\Rightarrow\; y = (1 - x^{2/3})^{3/2} \;\Rightarrow$

$\dfrac{dy}{dx} = \frac{3}{2}(1 - x^{2/3})^{1/2}\left(-\frac{2}{3}x^{-1/3}\right) = -x^{-1/3}(1 - x^{2/3})^{1/2} \;\Rightarrow$

$\left(\dfrac{dy}{dx}\right)^2 = x^{-2/3}(1 - x^{2/3}) = x^{-2/3} - 1$. Thus

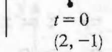


$L = 4\int_0^1 \sqrt{1 + (x^{-2/3} - 1)}\,dx = 4\int_0^1 x^{-1/3}\,dx = 4\lim\limits_{t\to 0^+}\left[\frac{3}{2}x^{2/3}\right]_t^1 = 6.$

$$= \frac{13\sqrt{13}-8}{27} + \frac{8}{27}\left(10\sqrt{10}-1\right) = \frac{13\sqrt{13}+80\sqrt{10}-16}{27}$$

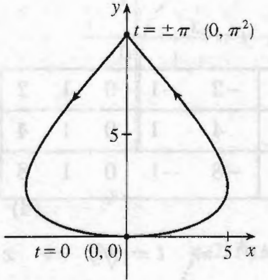
27. $y = 2x^{3/2} \quad \Rightarrow \quad y' = 3x^{1/2} \quad \Rightarrow \quad 1 + (y')^2 = 1 + 9x$. The arc length function with starting point $P_0(1, 2)$ is

$$s(x) = \int_1^x \sqrt{1+9t}\,dt = \left[\tfrac{2}{27}(1+9t)^{3/2}\right]_1^x = \tfrac{2}{27}\left[(1+9x)^{3/2} - 10\sqrt{10}\right].$$



3. $x = 5\sin t,\ y = t^2,\ -\pi \le t \le \pi$

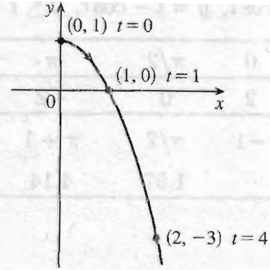
t	$-\pi$	$-\pi/2$	0	$\pi/2$	π
x	0	-5	0	5	0
y	π^2	$\pi^2/4$	0	$\pi^2/4$	π^2
	9.87	2.47		2.47	9.87



7. $x = \sqrt{t},\ y = 1 - t$

(a)

t	0	1	2	3	4
x	0	1	1.414	1.732	2
y	1	0	−1	−2	−3



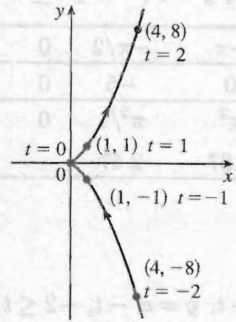
(b) $x = \sqrt{t} \quad \Rightarrow \quad t = x^2 \quad \Rightarrow \quad y = 1 - t = 1 - x^2.$

Since $t \geq 0$, $x \geq 0$.

8. $x = t^2,\ y = t^3$

(a)

t	−2	−1	0	1	2
x	4	1	0	1	4
y	−8	−1	0	1	8



(b) $y = t^3 \quad \Rightarrow \quad t = \sqrt[3]{y} \quad \Rightarrow \quad x = t^2 = \left(\sqrt[3]{y}\right)^2 = y^{2/3}.$

$t \in \mathbb{R}, y \in \mathbb{R}, x \geq 0.$

9. (a) $x = \sin\theta,\ y = \cos\theta,\ 0 \leq \theta \leq \pi.$

$x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1.$

Since $0 \leq \theta \leq \pi$, we have $\sin\theta \geq 0$, so $x \geq 0$. Thus, the curve is the right half of the circle $x^2 + y^2 = 1$.

(b)