Problem 7.2.10

$$\det[A - \lambda I_3] = (1 + \lambda)^2(1 - \lambda) = 0$$

Therefore, we have 2 eigenvalues, which are 1 with algebraic multiplicity 2, and -1 with algebraic multiplicity 1.

Problem 7.2.16

$$\det[A - \lambda I_2] = \lambda^2 - (a + c)\lambda + ac - b^2 = 0$$

if we want two distinct roots, then we have,

$$(a + c)^2 - 4(ac - b^2) > 0 \Longrightarrow (a - c)^2 + 4b^2 > 0$$

then we have $a \neq c$ and $b > 0$

Problem 7.2.18

$$\det[A - \lambda I_2] = \lambda^2 - 2a\lambda + a^2 - b^2 = 0$$

$\lambda = a \pm b$

Problem 7.2.22

$$\det[A - \lambda I] = \det[(A - \lambda I)^T] = \det[(A^T - \lambda I)]$$

which means, A and A^T have the same characteristic polynomials therefore, they have the same eigenvalues.