Problem 7.3.32

Compare the geometric multiplicities of A and A^T, we just compare the rank of $A - \lambda I$ and $A^T - \lambda I$. Because $n - rank(A - \lambda I)$ is the geometric multiplicity of λ in A.

And we know that $(A - \lambda I)^T = A^T - \lambda I^T = A^T - \lambda I$. And we know that a matrix has the same rank with its transpose. Therefore,

$$rank[A - \lambda I] = rank[A^T - \lambda I]$$

Hence, we get the geometric multiplicities of λ as an eigenvalue of A and A^T are the same.

Problem 7.3.34

(a) If $\overrightarrow{v} \in ker(B)$, then we have $B\overrightarrow{v} = 0$. Then $S^{-1}AS\overrightarrow{v} = 0 \Longrightarrow AS\overrightarrow{v} = S \times 0 = 0$. Therefore, $S\overrightarrow{v} \in ker(A)$.

(b) Because B and $S^{-1}AS$ have the same rank, which means the dimensions of $ker(B)$ and $ker(A)$ are the same. And for the linear transformation T, because S is an invertible matrix, therefore, we have that T is an isomorphism.

(c) Because S here is an invertible matrix, so it will not change the rank of matrix SH for any matrix H. So does S^{-1}. Therefore, $rank(B) = rank(A)$

Problem 7.3.36

The answer is No. because if two matrixes A and B are similar, then we can have a matrix S such that $B = S^{-1}AS$. Then $det(B) = det(S^{-1}AS) = det(S^{-1})det(A)det(S) = det(S^{-1})det(S)det(A) = \frac{1}{det(S)}det(S)det(A) = det(A)$

Here, for these two matrix, the det for the first one is -5 and for the second one is -3. So we have the answer.