110.201 Linear Algebra
HW11, middle 3

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7.4.12. The characteristic polynomial is $(\lambda-2)(\lambda-1)^2$. We have the eigenvector $\begin{bmatrix}1\\0\\0\end{bmatrix}$ associated with the eigenvalue 2 and the eigenvectors $\begin{bmatrix}1\\0\\1\end{bmatrix}$ and $\begin{bmatrix}0\\1\\0\end{bmatrix}$ associated with the eigenvalue 1. These are obviously linearly independent. So the geometric multiplicities equal the algebraic multiplicities, and both are equal to 3. So the matrix is diagonalizable, and can be written as $A = S^{-1}DS$, where D is the diagonal matrix with the eigenvalues on the diagonal, and $S = \begin{bmatrix}1 & 1 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$.

7.4.26. The characteristic polynomial of the matrix is obviously $(\lambda-2)(\lambda-1)^2$. However, the eigenspace associated with the eigenvalue 2 is $\begin{bmatrix}-a\\1\\0\end{bmatrix}$ and the eigenspace associated with the eigenvalue 1 is $\begin{bmatrix}1\\0\\0\end{bmatrix}$. So there are not enough eigenvectors, and the matrix is not diagonalizable.

7.4.38. Yes, since then A and B have the same characteristic polynomials, and hence the same eigenvalues, so that they are both similar to $\begin{bmatrix}\lambda_1 & 0\\0 & \lambda_2\end{bmatrix}$.