110.201 HW 11 solutions

May 9, 2005

pp. 350-354: 4, 6, 12.

(4) Let $z = re^{i\theta}$ with $r > 0$. Then it is easily verified that the two complex numbers $w_{\pm} = \pm\sqrt{r}e^{i\theta/2}$ satisfy $w^2 = z$. Uniqueness is guaranteed by the fundamental theorem of algebra: the polynomial $f(w) = w^2 - z$ can have at most two complex roots.

(6) We know that $z\bar{z}\ \|z\|^2$, so that for nonzero z, clearly $z\bar{z}/\|z\|^2 = 1$. Thus $z^{-1} = \bar{z}/\|z\|^2$. In polar form, letting $z = re^{i\theta}$, we obtain

$$z^{-1} = \frac{re^{-i\theta}}{r^2} = r^{-1}e^{-i\theta}.$$

I lack the LaTeXskills to do the diagram of the complex plane.

(12)

$$0 = \sum_{i=0}^{n} a_i\lambda^i = \bar{\sum_{i=0}^{n}} a_i\lambda^i = \sum_{i=0}^{n} \bar{a}_i\bar{\lambda}^i = \sum_{i=0}^{n} a_i\bar{\lambda}^i$$

because $a_i \in \mathbb{R}$.