

Problem 7.4.32

Yes, they are similar, because their eigenvalues are same.

Problem 7.4.48

The matrix for T is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

of course diagonalizable.

Problem 7.4.60

(a) Yes, they are eigenvectors of A .

Because

$$A(A - \lambda_1 I) = A^2 - \lambda_1 A = \lambda_2 A - \lambda_1 \lambda_2 I = \lambda_2 (A - \lambda_1 I)$$

That means, column vectors of matrix $A - \lambda_1 I$ are eigenvectors of A . For $A - \lambda_2 I$ same proof. And it works for all the 2 by 2 matrix.

As for $n \times n$ matrix, with two distinct eigenvalues. Suppose the algebraic multiplicity for λ_1 is s , for λ_2 is $n - s$. By problem 7.4.58, we have $(A - \lambda_1)(A - \lambda_2) = 0$. Then, we get

$$A(A - \lambda_1 I) = \lambda_2 (A - \lambda_1 I)$$

(b)

$$A^2 - (\lambda_1 + \lambda_2)A + \lambda_1 \lambda_2 I = 0$$

$$A(A - (\lambda_1 + \lambda_2)I) = -\lambda_1 \lambda_2 I$$

$$A\left(A - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\right) - \begin{bmatrix} \lambda_2 & 0 \\ 0 & \lambda_1 \end{bmatrix} = -\lambda_1 \lambda_2 I$$

$$A\left(A - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\right) = A \begin{bmatrix} \lambda_2 & 0 \\ 0 & \lambda_1 \end{bmatrix} - \lambda_1 \lambda_2 I$$

$$A\left(A - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\right) = \left(A - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\right) \begin{bmatrix} \lambda_2 & 0 \\ 0 & \lambda_1 \end{bmatrix}$$

End of proof.