110.201 Linear Algebra
HW12, middle 3

May 10, 2005

7.5.24. The characteristic polynomial is $x^3 - 3x^2 + 7x + 5$. The real root is approximately -0.557 and the complex conjugate roots are approximately $1.778 \pm 2.412i$.

7.5.28. We have that the characteristic polynomial has the form $(\lambda - 2)(\lambda^2 + a\lambda + b)$ with $a^2 - 4b < 0$, $\lambda_1 = 2$, and $\overline{\lambda_2} = \lambda_3$. Also, we have that $\lambda_1 + \lambda_2 + \lambda_3 = 8$ and $\lambda_1\lambda_2\lambda_3 = 50 = 2 \cdot 5^2$, so that $\lambda_2 + \lambda_3 = 8 - 2 = 6$ and $\lambda_2\lambda_3 = 25$. Using the subsitution $\lambda_3 = 6 - \lambda_2$ we get $\lambda_2\lambda_3 = \lambda_2(6 - \lambda_2) = 25$, or $\lambda_2 = 3 + 4i$. So $\lambda_3 = 3 - 4i$.

7.5.42. For all complex numbers z and w we have that $\overline{zw} = \overline{z}\,\overline{w}$. Applying this to each entry of AB yields the first part of the problem. Next, since A is real, we have that

$$\overline{A(\vec{v} + i\vec{w})} = A\overline{(\vec{v} + i\vec{w})} = \overline{(p + iq)}\,\overline{\vec{v} + i\vec{y}} = (p - iq)(\vec{v} - i\vec{y}).$$

This suffices.