

**Problem 8.1.22**

(a) If the matrix is symmetric, then it is diagonalizable. So we choose  $k = 2$

(b) Then we choose  $k = 0$  to let A to be non-diagonalizable.

**Problem 8.1.24**

The characteristic polynomial is  $(\lambda^2 - 1)^2$ . Then we have two eigenvalues 1 and  $-1$ .

Then,  $E_{-1} = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}\right)$  Make it orthonormal, then  $E_{-1} = \text{span}\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}\right)$ . And  $E_1 = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}\right)$ , make it orthonormal, then we have  $E_1 = \text{span}\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}\right)$ . Therefore, we have orthonormal basis which is

$$\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}\right)$$

**Problem 8.1.38**

(a) Its eigenvalue is 0, which algebraic multiplicity is n. then we have  $U^n = 0$

(b) It is actually a theorem, consider the characteristic polynomial  $\det(A - \lambda I) = (-\lambda)^n + \text{tr}A(-\lambda)^{n-1} \dots + \det(A) = (\lambda - \lambda_1) \dots (\lambda - \lambda_n)$ . Now we let  $\lambda$  be matrix A. We plug in then we get  $\det(A - A) = 0$ , therefore,  $(A - \lambda_1 I) \dots (A - \lambda_n I) = 0$ . Here, all the eigenvalues are 0, then we have  $A^n = 0$