

# Homework 1 Solutions

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pp. 5-8, 12, 14, 26, 32

12) Write down the augmented matrix and subtract twice the first row from the second. This gives a zero row, and shows that all  $(x, y) \in \mathbf{R}^2$  satisfying  $x - 2y = 3$  solve the linear system. Thus there are infinitely many solutions. Geometrically, this corresponds to two equations defining the same line in  $\mathbf{R}^2$ .

14) If  $R_1, R_2, R_3$  denote the three rows of the matrix defined by the system

$$x + 4y + z = 0$$

$$4x + 13y + 7z = 0$$

$$7x + 22y + 13z = 1$$

Note that  $2R_2 - R_1 = R_3$ . Performing the row operation  $R_1 - 2R_2 + R_3^*$  gives the equivalent system

$$x + 4y + z = 0$$

$$4x + 13y + 7z = 0$$

$$0x + 0y + 0z = 1$$

The last equation cannot be satisfied. Thus the system is inconsistent, and there are no solutions.

26) Subtract the first row from the second, and the first row from the third to obtain the system

$$x + y - z = 2$$

$$0x + y - 2z = 1$$

$$0x + 0y + (k^2 - 4)z = k - 2$$

The augmented matrix for this system is in reduced row echelon form (aside from a factor of  $k^2 - 4$  in the third row). It can be transformed into an upper triangular matrix with 1's on the main diagonal assuming that  $k^2 - 4 \neq 0$ , in which case the system will have a unique solution. If  $k = 2$ , the third row of the augmented matrix becomes a zero row, and since the two upper rows are obviously independent, the system will have infinitely many solutions. If  $k = -2$ , we will obtain an impossible equation (zero row = -4), and the system will have no solutions.

32) Letting  $f(x) = ax^2 + bx + c$ , the conditions given on  $f$  correspond to the system of linear equations

$$a + b + c = 1$$

$$4a + 2b + c = 0$$

$$(7a/3) + (3b/2) + c = -1$$

Using Gauss-Jordan elimination, substitution, or some other method, one finds that

$$a = 9, b = -28, c = 20$$

is the unique solution to the above equations.