1.2.6 This matrix is already in RREF-we don't need to do any row operations. The solutions are

$$x_1 = 3 + 7x_2 - x_5,\ x_3 = 2(1 + x_5),\ x_4 = 1 - x_5,\ x_2 = s,\ x_5 = t$$

where $s, t \in \mathbb{R}$.

1.2.18 The matrix in (a) is not in RREF. The second column contains two leading 1's. The matrix in (b) is in RREF.

1.2.24 Yes, there is. Let R_i and R_j denote two distinct but arbitrary rows in a matrix A, and a and b be two nonzero real numbers. A * indicates replacement of a row. Consider the elementary row operations:

i) $R_j \to aR_j^*$

ii) $R_i^* \iff R_j^*$

iii) $R_i \to bR_j + R_i^*$.

We can invert these transformations by applying the following elementary row operations:

i') $R_j \to a^{-1}R_j^*$

ii') $R_i^* \iff R_j^*$

iii') $R_i \to -bR_j + R_i^*$

1.2.28 Adding a multiple of one equation to another is an *invertible* transformation! By doing so you neither gain nor lose information, so the system has the same solutions. If you want a mathematical verification, let $x_1, \ldots x_n$ be real variables related to each other by two linear equations:

$$\sum_{j=1}^{n} a_j x_j = y_1$$

$$\sum_{j=1}^{n} b_j x_j = y_2$$

If c is a real number, we're going to subract c times the second equation from the first. Observe that this system implies the system (this is just substitution)

$$\left(\sum_{j=1}^{n} a_j x_j\right) - c\left(\sum_{j=1}^{n} b_j x_j\right) = \sum_{j=1}^{n}(a_j - cb_j)x_j = y_1 - cy_2$$

$$\sum_{j=1}^{n} b_j x_j = y_2,$$

i.e. the original system implies the second system. Conversely, the second system implies the original system by substituting $c\sum_j b_j x_j = cy_2$. Thus the two systems are equivalent, so the solutions are the same.