

# Linear Algebra Homework 2 Solutions

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1.3.4) Subtract twice row 1 from row 2 and thrice row one from row 3. This gives the matrix

$$\begin{pmatrix} 1 & 4 & 7 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix}$$

Now subtract twice row 2 from row 3 to see that the rank of the matrix is 2.

1.3.22) If a system of 3 equations in 3 variables  $x_1, \dots, x_3$  has a unique solution, we have equalities  $x = a, y = b, z = c$  for some real  $a, b, c$ . Thus the augmented matrix for the system can be reduced to the form

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array} \right]$$

It follows that the reduced row echelon form of  $A$  is the  $3 \times 3$  identity matrix.

1.3.34) (a) The value of  $A\vec{e}_i$  is the  $i$ th column of  $A$ ; thus  $A\vec{e}_1$  is equal to the vector  $[a \ d \ g]$ , written in row form, et cetera.

(b) Use the same principle in reverse;  $B\vec{e}_1 = \vec{v}_1$ , etc.

1.3.36) Use the result of 1.3.34 (a). The matrix is

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$