Week 2 HW

pp. 35-40; 44, 52

44. $A_{n\times m}$, $n > m$. Sketch: there can be at most m leading ones in $\text{rref}(A)$, so there are at least $n-m$ rows of zeros. If row i is all zeros, choose $b_i \neq 0$. Then $A\vec{x} = \vec{b}$ is inconsistent.

52. $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

$$A(B\vec{x}) = (AB)\vec{x} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\vec{x} = \begin{pmatrix} 0 & -1 \\ 2 & -1 \end{pmatrix}\vec{x} = C\vec{x}.$$

Take $C = \begin{pmatrix} 0 & -1 \\ 2 & -1 \end{pmatrix}$.

pp. 51-54; 4, 6

4. $T: \mathbb{R}^3 \to \mathbb{R}^4$. So the matrix A will be 4×3.

$$A = \begin{pmatrix} 9 & 3 & -3 \\ 2 & -9 & 1 \\ 4 & -9 & -2 \\ 5 & 1 & 5 \end{pmatrix}$$

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6. $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_2\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} x_1 + 4x_2 \\ 2x_1 + 5x_2 \\ 3x_3 + 6x_2 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$3\times 2 \quad 2\times 1$

Linear. The matrix is $A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$.