Week 3 Hw

P66 4, 6, 14, 26

4. $T(\vec{x}) = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\vec{x}$. $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

stretch and rotation

or, use fact 2.2.4, with $a=1$, $b=-1$. So, $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

and in polar coordinates, $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$, where

$r = \sqrt{(1)^2+(-1)^2} = \sqrt{2}$ and $\theta = \frac{7\pi}{4} = -\frac{\pi}{4}$. So we rotate

by $-\frac{\pi}{4}$, and scale by $\sqrt{2}$.

6. $L = \left\{ c\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle| c \in \mathbb{R} \right\}$. $\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Let $\vec{s}$ be a

vector in the direction of L, ~~then~~ say $\vec{s} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$.

Then $\text{proj}_L \vec{v} = \frac{\vec{v}\cdot\vec{s}}{\vec{s}\cdot\vec{s}}\vec{s} = \frac{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}\cdot\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \frac{5}{9}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$.

14. $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ is a unit vector in dir of line L. For $\vec{x} \in \mathbb{R}^3$,

(a) $\mathrm{proj}_L \vec{x} = (\vec{x} \cdot \vec{u})\vec{u} = \left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}\right]\vec{u} = (x_1u_1 + x_2u_2 + x_3u_3)\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$

$$= \begin{pmatrix} x_1u_1^2 + x_2u_1u_2 + x_3u_1u_3 \\ x_1u_1u_2 + x_2u_2^2 + x_3u_2u_3 \\ x_1u_1u_3 + x_2u_2u_3 + x_3u_3^2 \end{pmatrix} = \begin{pmatrix} u_1^2 & u_1u_2 & u_1u_3 \\ u_1u_2 & u_2^2 & u_2u_3 \\ u_1u_3 & u_2u_3 & u_3^2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= A\vec{x}.$$

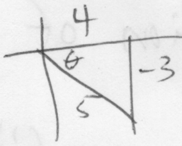
(b) {sum of diag} $=$ trace $A = u_1^2 + u_2^2 + u_3^2 = \vec{u} \cdot \vec{u} = \|\vec{u}\|^2$.

26. a) $4\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$. $A = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = 4I_2$.

b) $\begin{pmatrix} 2 \\ 3 \end{pmatrix} \to \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 0 \\ 5 \end{pmatrix} \to \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, so

$+ \sin\theta = \frac{-3}{5}$



Let

$\cos\theta = \frac{4}{5}$

$\theta = \arcsin\left(\frac{-3}{5}\right)$. Then $C = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, or

in other words, $\begin{pmatrix} 4/5 & 3/5 \\ -3/5 & 4/5 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 4 & 3 \\ -3 & 4 \end{pmatrix}$.

Week 3 HW

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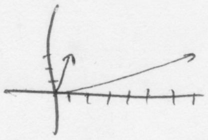
26d) $\begin{pmatrix}1\\3\end{pmatrix} \to \begin{pmatrix}7\\3\end{pmatrix}$.



horiz. shear.

$\begin{pmatrix}1&k\\0&1\end{pmatrix}\begin{pmatrix}1\\3\end{pmatrix}=\begin{pmatrix}7\\3\end{pmatrix}$, so $\begin{matrix}1+3k=7\\0+3=3\end{matrix}$, and $k=2$.

26e) $\begin{pmatrix}7\\1\end{pmatrix} \to \begin{pmatrix}-5\\5\end{pmatrix}$. $\begin{pmatrix}a&b\\b&-a\end{pmatrix}\begin{pmatrix}7\\1\end{pmatrix}=\begin{pmatrix}-5\\5\end{pmatrix}$

$\begin{cases}7a+b=-5\\7b-a=5\end{cases}$ or $\begin{pmatrix}7&1\\-1&7\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}-5\\5\end{pmatrix}$

so $a=-\frac{4}{5}$, $b=\frac{3}{5}$.