110.201 Homework 3 Solutions

March 1, 2005

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Problem 30 Just project $\mathbf{x}$ onto the line spanned by this vector. The answer will be parallel to the vector by definition. The formula for the projection onto the line L spanned by [1 2] is

$$\text{proj}_L\mathbf{x} = (\mathbf{x} \cdot \mathbf{u})\mathbf{u}$$

here $\mathbf{u}$ is the unit vector $\frac{1}{\sqrt{5}}\begin{bmatrix}1\\2\end{bmatrix}$.
Working this out in matrix form, we see that such a matrix A is given by

$$A = \begin{bmatrix}1/5 & 2/5\\2/5 & 4/5\end{bmatrix}$$

Problem 40 Let $\mathbf{u}_1$, $\mathbf{u}_2$ be unit vectors along the lines P and Q. Since they are perpendicular, they form a basis of $\mathbb{R}^2$. That means that each $\mathbf{x} \in \mathbb{R}^2$ has a unique representation

$$\mathbf{x} = a\mathbf{u}_1 + b\mathbf{u}_2$$

for some real numbers a and b. Take the dot product of this equation with both $\mathbf{u}_1$ and $\mathbf{u}_2$, use the orthogonality of $\mathbf{u}_1$ and $\mathbf{u}_2$, and conclude

$$\mathbf{x} = (\mathbf{x} \cdot \mathbf{u}_1)\mathbf{u}_1 + (\mathbf{x} \cdot \mathbf{u}_2)\mathbf{u}_2$$

$$= \text{proj}_P\mathbf{x} + \text{proj}_Q\mathbf{x}$$

Thus $(\text{proj}_P + \text{proj}_Q)\mathbf{x} = \mathbf{x}$.

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Problem 28 Set up the problem in the usual way; we need to reduce the matrix

$$\left[\begin{array}{cccc|cccc} 22 & 13 & 8 & 3 & 1 & 0 & 0 & 0 \\ -16 & -3 & -2 & -2 & 0 & 1 & 0 & 0 \\ 8 & 9 & 7 & 2 & 0 & 0 & 1 & 0 \\ 5 & 4 & 3 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

to reduced row echelon form. The key here is to refrain from introducing fractions! This is a standard procedure and I'll just state the answer. It is

$$A^{-1} = \begin{bmatrix} 1 & -2 & 9 & -25 \\ -2 & 5 & -22 & 60 \\ 4 & -9 & 41 & -112 \\ -9 & 17 & -80 & 222 \end{bmatrix}$$

Problem 38 Write out the formula for the inverse of a 2×2 matrix:

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} -1 & -k \\ 0 & 1 \end{bmatrix} = A$$

Be sure to check this.