Problem No. 2.3.42 Solution

the permutation matrix is invertible, because for a $n \times n$ permutation matrix A. The rank of A is n. You can easily get the rref(A) and find it to be the identity matrix I_n. For the second question. the answer is YES. Cuz the only thing we do to find the inverse matrix of permutation is to switch rows. We do not do addition, scaling. That means its inverse is still a permutation matrix.

Problem 2.4.34. Solution

Now suppose we have two $n \times n$ matrices A,B. and AB is invertible.then $rank(AB) = n$. As for AB, $rank(AB) = \min(rank(A), rank(B))$, and we know that $rank(A) \leq n$ and $rank(B) \leq n$. That means both rank(A) and rank(B) are n. They are invertible.

Problem 2.4.36 Solution

Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and we know that $AX = 0$. so we can get $a = -2c$, $b = -2d$. therefore $X = \begin{pmatrix} -2c & -2d \\ c & d \end{pmatrix}$ where c, d are random numbers.

Problem 2.4.40 Solution

We have $(AB)^{-} = B^{-}A^{-1}$, and we know that $B^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, then, $B = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$. So we have $B(AB)^{-1} = A^{-1}$. Then we have $A^{-1} = \begin{pmatrix} -1 & -5 \\ 1 & 4 \end{pmatrix}$ so $A = \begin{pmatrix} 4 & 5 \\ -1 & -1 \end{pmatrix}$