Linear Alg. HW 5

4.1.28 $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} a & a+b \\ c & c+d \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ c & d \end{pmatrix}$, so

$a = a+c \qquad c+d = d \qquad a+b = b+d$

$0 = c \qquad c = 0 \qquad a = d$

So $A = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} = aI + b\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \equiv aI + bJ$.

So a basis is $\{I, J\}$.

4.1.36 Let $A = \begin{pmatrix} a & b & c \\ d & e & f \\ h & i & j \end{pmatrix}$. Then $AB = BA$ says

$$\begin{pmatrix} 2a & 3b & 3c \\ 2d & 3e & 3f \\ 2g & 3h & 3i \end{pmatrix} = \begin{pmatrix} 2a & 2b & 2c \\ 3d & 3e & 3f \\ 3g & 3h & 3i \end{pmatrix}$$

However, the circled places represent unsolvable equations (eg, $2d = 3d \Rightarrow 2 = 3$, for $d \neq 0$). So the space consists only of the zero matrix, $O_{3\times3}$.

4.2.28 $T(f(t)) = f(2t) - f(t) : P_2 \to P_2$

$f(t) = a + bt + ct^2 \in P_2$ is a typical element, so $f(2t) = a + 2bt + 4ct^2$,
and $f(2t) - f(t) = (a-a) + (2b-b)t + (4c-c)t^2$
$= bt + 3ct^2$

Let $g = d + et + ft^2$.

$T(f+g) = T[(a+d) + (b+e)t + (c+f)t^2]$

$= (b+e)t + 3(c+f)t^2$
$= (bt + 3ct^2) + (et + 3ft^2) = T(f) + T(g)$.

$T(kf) = T(ka + kbt + kct^2)$
$= kbt + 3kct^2$
$= k(bt + 3ct^2) = kT(f)$. Linear.

$T(1) = T(1 + 0t + 0t^2) = 0t + 3 \cdot 0t^2 = 0$, so
$\ker T \neq \{0\}$. Not an isomorphism.

4.2.40 $T(f) = f'' + 2f' + f : C^\infty \to C^\infty$

$T(f+g) = (f+g)'' + 2(f+g)' + (f+g) = f'' + g'' + 2f' + 2g' + f + g$
$= f'' + 2f' + f + g'' + 2g' + g$
$= T(f) + T(g)$

$T(kf) = (kf)'' + 2(kf)' + kf = k(f'' + 2f' + f) = kT(f)$.

The solns of $f'' + 2f' + f = 0$ form a two dimensional subspace of C^∞ by fact 4.1.7, so $\ker T \neq \{0\}$.