110.201 Homework 5 Solutions

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(16) If the vectors \( \vec{v}_1, \vec{v}_2, \vec{v}_3 \) are linearly independent, they will span \( \mathbb{R}^3 \), so that \( \vec{x} \) will automatically be in their span. A quick row reduction exercise shows that \( \vec{v}_1, \vec{v}_2, \vec{v}_3 \) are linearly independent, so that \( \vec{x} \) does lie in their span.

(30) If you really wanted to, you could go ahead and compute \( S \) and change basis that way. But this one can, thankfully, be done in a much easier way. Some quick computation shows that

\[
A \vec{v}_1 = \vec{v}_1, \quad A \vec{v}_2 = -\vec{v}_2, \quad A \vec{v}_3 = \vec{0}
\]

It follows immediately that the matrix of the linear transformation represented by \( A \) in the basis \( \mathcal{B} \) is

\[
B = \begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(46) We are looking for linearly independent vectors \( \vec{v}_1, \vec{v}_2 \) in the given plane such that \( \vec{x} = 2\vec{v}_1 - \vec{v}_2 \). By fooling around, you can find that two such vectors are given by \( \vec{v}_1 = (1, -\frac{1}{4}, 0), \vec{v}_2 = (0, \frac{1}{2}, -1) \). A more systematic way to do the problem would go as follows: Find two linearly independent vectors in \( \mathbb{R}^3 \) such that

1. \( \vec{x} = 2\vec{v}_1 - \vec{v}_2 \)
2. \( \text{proj}_V \vec{v}_1 \) and \( \text{proj}_V \vec{v}_2 \) are linearly independent as well.

This is easy to do by just messing around a bit. Then the projections of \( \vec{v}_1 \) and \( \vec{v}_2 \) will span \( V \), and we will have the equation
2\text{proj}_V\vec{v}_1 - \text{proj}_V\vec{v}_2 = \text{proj}_V\vec{x} = \vec{x}

solving the problem.

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(8) Yes, this is a subspace of \( \mathbb{R}^9 \). It consists of all matrices of the form

\[
\begin{bmatrix}
* & * & * \\
0 & * & * \\
0 & 0 & *
\end{bmatrix}
\]

Here the *'s denote entries that may be nonzero. It is clear that the sum of any two such matrices is also upper triangular. So is any scalar multiple of such a matrix. The zero matrix is upper triangular, so all the axioms for a subspace are satisfied.