110.201 Homework 5 Solutions

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(16) If the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent, they will span $\mathbb{R}^3$, so that $\vec{x}$ will automatically be in their span. A quick row reduction exercise shows that $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent, so that $\vec{x}$ does lie in their span.

(30) If you really wanted to, you could go ahead and compute S and change basis that way. But this one can, thankfully, be done in a much easier way. Some quick computation shows that

$$A\vec{v}_1 = \vec{v}_1,\ A\vec{v}_2 = -\vec{v}_2,\ A\vec{v}_3 = \vec{0}$$

It follows immediately that the matrix of the linear transformation represented by A in the basis $\mathcal{B}$ is

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(46) We are looking for linearly independent vectors $\vec{v}_1, \vec{v}_2$ in the given plane such that $\vec{x} = 2\vec{v}_1 - \vec{v}_2$. By fooling around, you can find that two such vectors are given by $\vec{v}_1 = (1, -\frac{1}{4}, 0)$, $\vec{v}_2 = (0, \frac{1}{2}, -1)$. A more systematic way to do the problem would go as follows: Find two linearly independent vectors in $\mathbb{R}^3$ such that

(1) $\vec{x} = 2\vec{v}_1 - \vec{v}_2$

(2) $\text{proj}_V \vec{v}_1$ and $\text{proj}_V \vec{v}_2$ are linearly independent as well.

This is easy to do by just messing around a bit. Then the projections of $\vec{v}_1$ and $\vec{v}_2$ will span V, and we will have the equation

$$2\text{proj}_V\vec{v}_1 - \text{proj}_V\vec{v}_2 = \text{proj}_V\vec{x} = \vec{x}$$

solving the problem.

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(8) Yes, this is a subspace of $\mathbb{R}^9$. It consists of all matrices of the form

$$\begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}$$

Here the *'s denote entries that may be nonzero. It is clear that the sum of any two such matrices is also upper triangular. So is any scalar multiple of such a matrix. The zero matrix is upper triangular, so all the axioms for a subspace are satisfied.