Linear Algebra
HW 6
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$\boxed{3.4.28}$ $B = \left[ T(\vec{v}_1)_B \quad T(\vec{v}_2)_B \quad T(\vec{v}_3)_B \right]$

$T(\vec{v}_1) = A\vec{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\qquad T(\vec{v}_1)_B = 0\vec{v}_1 + 0\vec{v}_2 + 0\vec{v}_3$

$T(\vec{v}_2) = A\vec{v}_2 = \begin{pmatrix} 9 \\ -9 \\ 0 \end{pmatrix}$ $\qquad T(\vec{v}_2)_B = 0\vec{v}_1 + 9\vec{v}_2 + 0\vec{v}_3$

$T(\vec{v}_3) = A\vec{v}_3 = \begin{pmatrix} 0 \\ 9 \\ -18 \end{pmatrix}$ $\qquad T(\vec{v}_3)_B = 0\vec{v}_1 + 0\vec{v}_2 + 9\vec{v}_3$

so $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$.

$\boxed{3.4.34}$ $\mathfrak{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, $\vec{v}_3 = \vec{v}_1 \times \vec{v}_2$.

$B = \left[ T(\vec{v}_1)_B \quad T(\vec{v}_2)_B \quad T(\vec{v}_3)_B \right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-2\|v_3\|^2 \end{pmatrix}$.

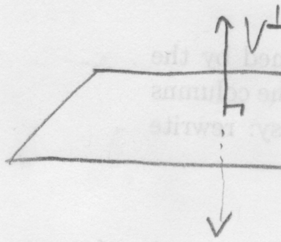
$T(\vec{v}_1) = \vec{v}_1 - 2(\vec{v}_3 \cdot \vec{v}_1)\vec{v}_3 = \vec{v}_1$

$T(\vec{v}_2) = \vec{v}_2 - 2(\vec{v}_3 \cdot \vec{v}_2)\vec{v}_3 = \vec{v}_2$

$T(\vec{v}_3) = \vec{v}_3 - 2(\vec{v}_3 \cdot \vec{v}_3)\vec{v}_3 = \vec{v}_3 - 2\|\vec{v}_3\|^2\vec{v}_3 = (1 - 2\|\vec{v}_3\|^2)\vec{v}_3$

This is almost reflection about $L^\perp$, where $L = \text{span}\{\vec{v}_3\}$.
But $\vec{v}_3$ is not a unit vector.

$\boxed{3.4.42}$ reflection about $x_1 - 2x_2 + 2x_3 = 0$ in $\mathbb{R}^3$.




Let $\vec{u} = \frac{1}{3}\begin{pmatrix}1\\-2\\2\end{pmatrix}$.

Then $\mathrm{refl}_V \vec{x} = \vec{x} - 2(\vec{x}\cdot\vec{u})\vec{u} \qquad \forall x \in \mathbb{R}^3$

$$= \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} - \frac{2}{3}(x_1 - 2x_2 + 2x_3)\frac{1}{3}\begin{pmatrix}1\\-2\\2\end{pmatrix} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} - \frac{2}{9}\begin{pmatrix}x_1 - 2x_2 + 2x_3\\ -2x_1 + 4x_2 - 4x_3\\ 2x_1 - 4x_2 + 4x_3\end{pmatrix}$$

$$= \begin{pmatrix}\frac{7}{9}x_1 + \frac{4}{9}x_2 - \frac{4}{9}x_3\\ \frac{4}{9}x_1 + \frac{1}{9}x_2 + \frac{8}{9}x_3\\ -\frac{4}{9}x_1 + \frac{8}{9}x_2 + \frac{1}{9}x_3\end{pmatrix} = \frac{1}{9}\begin{pmatrix}7 & 4 & -4\\ 4 & 1 & 8\\ -4 & 8 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$= A\vec{x} = T(\vec{x}).$$

$$[T(\vec{x})]_B = \left[\, T(\vec{v}_1)_B \;\; T(\vec{v}_2)_B \;\; T(\vec{v}_3)_B \,\right].$$

$$= \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \text{ if } \vec{v}_1 = \frac{1}{9}\begin{pmatrix}7\\4\\-4\end{pmatrix}, \vec{v}_2 = \frac{1}{9}\begin{pmatrix}4\\1\\8\end{pmatrix}, \vec{u}_3 = \begin{pmatrix}-4\\8\\1\end{pmatrix}.$$

$\boxed{3.4.62}$ $T(\vec{x}) = \begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}\vec{x}$. Let $\{\vec{v}, \vec{w}\}$ be the required basis.

$$B = \begin{pmatrix}5 & 0\\ 0 & 1\end{pmatrix} = \left[\, [T(\vec{v})]_B \quad [T(\vec{w})]_B \,\right]$$

$$T(\vec{v}) = \begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}\begin{pmatrix}v_1\\ v_2\end{pmatrix} = \begin{pmatrix}v_1 + 2v_2\\ 4v_1 + 3v_2\end{pmatrix} = \begin{pmatrix}5\\ 0\end{pmatrix} \Rightarrow \vec{v} = \begin{pmatrix}v_1\\ v_2\end{pmatrix} = \begin{pmatrix}-3\\ 4\end{pmatrix}$$

$$T(\vec{w}) = \begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}\begin{pmatrix}w_1\\ w_2\end{pmatrix} = \begin{pmatrix}w_1 + 2w_2\\ 4w_1 + 3w_2\end{pmatrix} = \begin{pmatrix}0\\ 1\end{pmatrix} \Rightarrow \vec{w} = \begin{pmatrix}w_1\\ w_2\end{pmatrix} = \frac{1}{5}\begin{pmatrix}2\\ -1\end{pmatrix}$$