Problem 4.2.80

Suppose those students' name are $A_1, A_2, \cdots, A_n$, and Let the linear space Y be R^n with standard basis $\{e_1, e_2, \cdots e_n\}$. Now we define a linear operator:

$$T : A_i \dashrightarrow e_i$$

Because actually those students are independent, so we can consider them as basis, therefore, the linear operator is mapping basis to basis. then we get the answer.

Problem 4.3.16

By fact 4.3.2, we get

$$B = [[T(1+i)]_{\mathbb{B}}, [T(1-i)]_{\mathbb{B}}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Then the rank of B is 2, T is isomorphic.

Problem 4.3.28

By the same way, we get

$$B = [[T(1)]_{\mathbb{B}}, [T(t-1)]_{\mathbb{B}}, [T((t-1)^2)]_{\mathbb{B}}] = [[1]_{\mathbb{B}}, [2t-2]_{\mathbb{B}}, [4(t-1)^2]_{\mathbb{B}}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Then the rank of B is 3, T is isomorphic.

Problem 4.3.32

By the same way as above, we get

$$B = [[T(1)]_{\mathbb{B}}, [T(t)]_{\mathbb{B}}, [T(t^2)]_{\mathbb{B}}] = [[1]_{\mathbb{B}}, [t]_{\mathbb{B}}, [2t-1]_{\mathbb{B}}] = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Not isomorphic, and basis of kernel of matrix B is

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

which is $(t-1)^2$. And basis of image is $\{1, t\}$ therefore the rank is 2.