110.201 Linear Algebra
Homework 7 Solutions
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5.1.10 $\vec{u} \cdot \vec{v} = 2 + 3k + 4 = 0$, so $k = -2$.

5.1.12 We have

$||\vec{v}+\vec{w}||^2 = (\vec{v}+\vec{w})\cdot(\vec{v}+\vec{w}) \leq |\vec{v}\cdot\vec{v}|+2|\vec{v}\cdot\vec{w}|+|\vec{w}\cdot\vec{w}| \leq ||\vec{v}||^2+2||\vec{v}||||\vec{w}||+||\vec{w}||^2 = (||\vec{v}||+||\vec{w}||)^2$.

Now take the square root of both sides.

5.1.16 So, if $\vec{u}_4$ is orthogonal to everything else, then we must have that $\vec{u}_4 \cdot \vec{u}_i = 0$ for $i = 1, 2, 3$. In other words, if $\vec{u}_4 = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, then we have

$$a + b + c + d = 0 \quad (1)$$

$$a + b - c - d = 0 \quad (2)$$

$$a - b + c - d = 0. \quad (3)$$

We know $\vec{u}_4$ must be some permutation of positive and negative 1/2. The first equation says that two of the a, b, c, d must be positive and the other two must be negative. Adding the last two equations, we get that $a = d$, and subtracting them, we get that $b = c$. Normalizing, this says that $\vec{u}_4 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$ has the required properties.

5.1.28 Let V be the given subspace, and $\vec{x}$ the vector to be orthogonally projected. Normalize the basis vectors by multiplying by 1/2. We now have that

$$\text{proj}_V(\vec{x}) = (\vec{u_1} \cdot \vec{x})\vec{u_1} + (\vec{u}_2 \cdot \vec{x})\vec{u}_2 + (\vec{u}_3 \cdot \vec{x})\vec{u}_3 = \frac{1}{2}\begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}.$$