Problem 5.3.30

The Kernel of L is $\vec{0}$. The dimension of the image is m. and so $m \leq n$. If A is the matrix of L, then A is an orthogonal matrix, that means $A^T A = I_m$ and AA^T is a matrix of an orthogonal projection. The example is let A be

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

And

$$T : \begin{bmatrix} a \\ b \end{bmatrix} \dashrightarrow \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

Problem 5.3.32

(a) The answer is No, by fact 5.3.8 in the book, we have A is an orthogonal matrix, and by fact 5.3.10, we have AA^T must be a orthogonal projection. Now n is not equal to m, and if $AA^T = I_n$, we know that $rank(AA^T) \leq rank(A) = rank(A^T)$, and $rank(A^T A) \leq rank(A) = rank(A^T)$ therefore $n \leq rank(A)$, $m \leq rank(A)$. Now A is a n by m matrix, we have $rank(A)$ must be less than m and n, contradiction.

(b) The answer is Yes, because $A^T A = I$, then $A^T = A^{-1}$, so $AA^T = I$.

Problem 5.3.42

(a) Well, A^2 is to do orthogonal projection twice, then for the second time, nothing changed, therefore, $A^2 = A$

b Because A is a matrix of an orthogonal projection, we have $A = QQ^T$, where Q is an orthogonal matrix. Then $A^2 = QQ^T QQ^T = QQ^T = A$

Problem 5.3.48

By hint, we do QR-factorization for A^T, then $A^T = SR$, where S is an orthogonal matrix, and R is an upper triangular matrix. $A = R^T S^T$. Let L be R^T, so L is an lower triangular matrix, and let Q be S^T, then Q is an orthogonal matrix.