Homework 9 Solutions: pp. 271-274

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(4) Use the steps (the first row listed in each entry is to be modified)

1. (II) + (I)
2. (III) - 2(I)
3. (III) - 3(II)
4. (IV) +2(I) - 4(II) - 2(III)

to reduce A to an upper triangular matrix with diagonal elements 1, 1, 1 and 9 respectively. Therefore $\det A = 9$

(18) With respect to the standard basis $\mathcal{B} = (1, t, t^2)$, the linear transformation $f(t) \mapsto f(3t-2)$ acts as follows:

$$1 \mapsto 1 \quad t \mapsto 3t-2 \quad t^2 \mapsto 9t^2 - 6t + 4$$

and so its matrix is

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 4 & -6 & 9 \end{pmatrix}$$

and so its determinant is $3 \times 7 \times 9$.

(30) (a) If you expand along the first column it is obvious that no terms of order higher than 2 in t appear. The coefficient of t^2 can be determined straightforwardly by whatever method you like to be $b - a$.

(b) If $t = a, b$ we see immediately that the matrix defining $f(t)$ will have two (or more) equal columns, which implies that its determinant is zero. Many people did this by plugging into the polynomial they obtained in part (a). This is much faster.

(c) Since $f(t)$ is a quadratic function and has two distinct roots a and b, $f(t)$ is nonzero for all other t. So the matrix is invertible for $t \neq a, b$.

(40) Since A is orthogonal, $A^{-1} = A^t$. So we have

$$\det(AA^{-1}) = 1 = \det(AA^t) = (\det A)^2.$$

It is now clear that $\det A = \pm 1$