Mathematics 110.201: Linear Algebra. Spring 2002

Midterm exam. 8 April 2002.

Name____

TA_______________________ ***or*** Section____________________________

Please SIGN the following statement:
"I am a student enrolled in the course and the name above is my true full name."

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Your exam will not be graded unless you EITHER sign the above statement OR positively identify yourself to the proctor.

Instructions: No notes or calculator may be used. *Please explain the procedure followed to reach your answers.* The proctor is qualified to answer questions about the statement of the problems *only*.

Time: 50 minutes.

Question 1 (20 points)	
Question 2 (20 points)	
Question 3 (20 points)	
Question 4 (20 points)	
Question 5 (20 points)	
TOTAL	

1. Find the QR factorization of the matrix

$$\begin{bmatrix} 4 & 25 \\ 0 & 0 \\ 3 & -25 \end{bmatrix}.$$

2. For the linear system $A\vec{x} = \vec{b}$ with

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

find:

(a) the normal equation,

(b) the least squares solution (or solutions) of the system,

(c) the projection $\vec{b}_*$ of $\vec{b}$ onto $\mathrm{Im}(A)$, and

(d) the matrix B of the projection from $\mathbb{R}^4$ onto $\mathrm{Im}(A)$. (*Help:* after you find B check that it is a symmetric matrix. It should.)

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3. Consider the parallelogram $\mathcal{P}$ in $\mathbb{R}^3$ defined by the vectors

$$\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$$

Compute the area of $\mathcal{P}$ in TWO different ways. (We have studied at least three different ways of finding the area!) [*Help:* Observe that $\vec{u} \cdot \vec{v} = 0$.)

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4. Is the following statement true or false? Justify your answer.

"The rule

$$\langle \vec{v}, \vec{w} \rangle = \|\vec{v} \times \vec{w}\| \tag{1}$$

defines an inner product in $\mathbb{R}^3$ (though possibly different from the standard dot product in $\mathbb{R}^3$)."

(In equation (1), $\|\vec{v} \times \vec{w}\|$ denotes the usual length of the vector $\vec{v} \times \vec{w}$.)

5. Recall that $C[-\pi, \pi]$ is an inner product space with

$$\langle f, g\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)g(t)dt.$$

The space P_1 of linear polynomials (actual polynomials, not trigonometric polynomials) is a subspace of $C[-\pi, \pi]$. Find the projection of the sine function $\sin(t)$ onto the subspace P_1 by following the steps below.

(a) Apply the Gram-Schmidt process to the basis $\mathfrak{B} = \{1, t\}$ of P_1 in order to obtain an orthonormal basis $\mathfrak{U}$ of P_1.

(b) Use the orthonormal basis $\mathfrak{U}$ to compute the desired projection (this projection is the linear polynomial that best approximates the sine function in the least-squares sense.)
Help: $\int_{-\pi}^{\pi} t \sin t \, dt = 2\pi$.

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