(18) The column vectors of a 5×4 matrix must be linearly dependent.

T F no ~~all zero for example~~,

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(19) The system $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is inconsistent for all 4×3 matrices A.

T F

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ for example

(20) If $A^2 = I_n$ then the matrix A must be invertible.

T F True for any $AB = I_n$ A square.

(21) If $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ and $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m$ are any two bases of a subspace V of $\mathbb{R}^{10}$ then n must be equal to m.

T F basis give dim

(22) If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^4$ then $\vec{v}$ is a linear combination of $\vec{v}$ and $\vec{w}$.

T F $v = 1 \cdot v + 0w$

(23) If the matrix A is invertible then the matrix $5A$ is also invertible.

T F $(5A)^{-1} = \frac{1}{5} A^{-1}$

(24) If the kernel of a matrix A consists of the zero vector only then the column vectors of A must be linearly independent.

T F $Ax = x_1 v_1 + \cdots x_n v_n = 0$

if lin ind x's $= 0$.

(25) If $\vec{v}, \vec{u}$, and $\vec{w}$ are vectors in $\mathbb{R}^2$ then $\vec{w}$ must be a linear combination of $\vec{v}$ and $\vec{u}$.

T F if $v = u = 0$ $w \neq 0$

(26) The matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is invertible.

T F rank $= 1$

(27) If the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ and $\vec{v}_4$ are linearly independent then so are the vectors $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$

T F same reason