Exam #1, Linear Algebra, Spring, 2003, W. Stephen Wilson

Name: ________________________________

TA Name and section: ________________________________
(1 point for recognizability and 1 point for spelling)

NO CALCULATORS. KEEP IT OFF YOUR DESK. All questions are true-false questions. The scoring is as follows: Wrong answer, 0 points; No answer, 1 point; Correct answer, 2 points; Correct answer with correct reason, 3 points. If you can't answer something in less than a minute, move on. Use very very short, trivial, nearly frivolous (but true) reasons. Say anything true.

(1) A system of 4 equations in 3 unknowns is always inconsistent.

T **F**

(2) If A is any invertible $n \times n$ matrix then $\text{rref}(A) = I_n$.

T **F**

(3) The image of a 3×4 matrix is a subspace of $\mathbb{R}^4$.

T **F**

(4) There is a 3×4 matrix of rank 4.

T **F**

(5) The formula $(A^2)^{-1} = (A^{-1})^2$ holds for all invertible matrices A.

T **F**

(6) The span of the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ consists of all linear combinations of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$.

T **F**

(7) There exists a system of 3 linear equations in 3 unknowns with exactly 3 solutions.

T **F**

(8) The formula $AB = BA$ holds for all $n \times n$ matrices A and B.

T **F**

(9) If the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are linearly independent in $\mathbb{R}^n$ then they must form a basis for $\mathbb{R}^n$.

T **F**

(10) There is a 5×5 matrix A with rank 4 such that the system $A(\vec{x}) = \vec{0}$ has only the solution $\vec{x} = \vec{0}$.

T **F**

(11) If A is a 3×4 matrix and B is a 4×5 matrix then AB is a 5×3 matrix.

T **F**

(12) If $2\vec{u} + 3\vec{v} + 4\vec{w} = 5\vec{u} + 6\vec{v} + 7\vec{w}$ then the vectors $\vec{u}, \vec{v}$, and $\vec{w}$ must be linearly dependent.

T **F**

(13) There is a matrix A such that $A \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$.

T **F**

(14) The function $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ 1 \end{bmatrix}$ is a linear transformation.

T **F**

(15) The kernel of any invertible matrix consists of the zero vector only.

T **F**

(16) The system $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is inconsistent.

T **F**

(17) There is an invertible matrix A such that $A^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

T **F**

(18) The column vectors of a 5×4 matrix must be linearly dependent.

T **F**

(19) The system $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is inconsistent for all 4×3 matrices A.

T **F**

(20) If $A^2 = I_n$ then the matrix A must be invertible.

T **F**

(21) If $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ and $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_m$ are any two bases of a subspace V of $\mathbb{R}^{10}$ then n must be equal to m.

T **F**

(22) If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^4$ then $\vec{v}$ is a linear combination of $\vec{v}$ and $\vec{w}$.

T **F**

(23) If the matrix A is invertible then the matrix $5A$ is also invertible.

T **F**

(24) If the kernel of a matrix A consists of the zero vector only then the column vectors of A must be linearly independent.

T **F**

(25) If $\vec{v}, \vec{u}$, and $\vec{w}$ are vectors in $\mathbb{R}^2$ then $\vec{w}$ must be a linear combination of $\vec{v}$ and $\vec{u}$.

T **F**

(26) The matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is invertible.

T **F**

(27) If the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ and $\vec{v}_4$ are linearly independent then so are the vectors $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$

(28) If the vector $\vec{u}$ is a linear combination of $\vec{v}$ and $\vec{w}$ then the vector $\vec{w}$ must be a linear combination of $\vec{u}$ and $\vec{v}$.

T **F**

(29) If matrices A and B are both invertible then the matrix $A + B$ must be invertible too.

T **F**

(30) The vectors of the form $\begin{bmatrix} a \\ b \\ 0 \\ a \end{bmatrix}$ (where a and b are arbitrary real numbers) form a subspace of $\mathbb{R}^4$.

T **F**

(31) There exists a 2×2 matrix A such that $A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $A \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

T **F**

(32) There is a matrix A such that $A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$.

T **F**

(33) If a subspace V of $\mathbb{R}^n$ contains none of the standard vectors $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$ then V consists of the zero vector only.

T **F**

(34) If A and B are any two 3×3 matrices of rank 2 then A can be transformed into B by means of elementary row operations.

T **F**

(35) $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}^3 = \begin{bmatrix} 1 & 3k \\ 0 & 1 \end{bmatrix}$ for all real numbers k.

T **F**

(36) If a subspace V of $\mathbb{R}^3$ contains the standard vectors $\vec{e}_1, \vec{e}_2$, and $\vec{e}_3$ then V must be $\mathbb{R}^3$.

T **F**

(37) If A is a 4×3 matrix then there exists a vector $\vec{b}$ in $\mathbb{R}^4$ such that the system $A\vec{x} = \vec{b}$ is inconsistent.

T **F**

(38) There is an invertible $n \times n$ matrix with two identical rows.

T **F**

(39) If $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$ are any 3 vectors in $\mathbb{R}^3$ then there must be a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that $T(\vec{v}_1) = \vec{e}_1$, $T(\vec{v}_2) = \vec{e}_2$ and $T(\vec{v}_3) = \vec{e}_3$.

T **F**

(40) If A is a 4×3 matrix of rank 3 and $A\vec{v} = A\vec{w}$ for two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$ then the vectors $\vec{v}$ and $\vec{w}$ must be equal.

T **F**

(41) If A and B are two 4×3 matrices such that $A\vec{v} = B\vec{v}$ for all vectors $\vec{v}$ in $\mathbb{R}^3$ then the matrices A and B must be equal.

T **F**

(42) If V is any 3-dimensional subspace of $\mathbb{R}^5$ then V has infinitely many bases.

T **F**

(43) If $A = \begin{bmatrix} \vec{u} & \vec{v} & \vec{w} \end{bmatrix}$ and $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ then the equation $\vec{w} = 2\vec{u} + 3\vec{v}$ must hold.

T **F**

(44) The formula $(A\vec{v}) \cdot (A\vec{w}) = \vec{v} \cdot \vec{w}$ holds for all invertible 2×2 matrices A and for all vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^2$.

T **F**

(45) If $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ and $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_n$ are two bases of $\mathbb{R}^n$ then there is a linear transformation T from $\mathbb{R}^n$ to $\mathbb{R}^n$ such that $T(\vec{v}_1) = \vec{w}_1, T(\vec{v}_2) = \vec{w}_2, \ldots, T(\vec{v}_n) = \vec{w}_n$.

T **F**