110.202. Linear Algebra
2003 Summer
Midterm II
6/17/2003 1:00PM

1. (20pts) (a) Find an orthonormal basis of the space P_1 with inner product

$$\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt.$$

(b) Find the linear polynomial $g(t) = a + bt$ that best approximates the function $f(t) = t^2$ in the interval $[0, 1]$ in the (continuous) least-squares sense.

2. (10pts) Consider the subspace W of $\mathbb{R}^4$ spanned by the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} 1 \\ 9 \\ -5 \\ 3 \end{bmatrix}.$$

Find the matrix of the orthogonal projection onto W.

3. (10pts) Use Cramer's rule to solve the system

$$\begin{cases} x_1 & & + & x_3 & = & 1 \\ 2x_1 & - & 4x_2 & + & 5x_3 & = & 0 \\ & - & 2x_2 & - & x_3 & = & 4 \end{cases}.$$

4. (10pts) Find the determinant of

$$A = \begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 0 & 3 & 1 \\ 3 & 2 & 0 & 1 \\ 0 & -1 & 7 & 2 \end{bmatrix}.$$

5. (10pts) Find the trigonometric function of the form

$$f(t) = c_0 + c_1 \sin(t) + c_2 \cos(t)$$

that best fits the data points $(0, -1)$, $\left(\frac{\pi}{2}, 2\right)$, $(\pi, 2)$ and $\left(\frac{3\pi}{2}, 1\right)$, using lease squares.

6. (20pts) Find the QR factorization of

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -2 \\ -1 & 1 & 0 \end{bmatrix}.$$

7. (10pts) Find the matrix of the linear transformation,

$$T(M) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} M$$

from $M_{2\times 2}$ to $M_{2\times 2}$ with respect to the basis,

$$\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$$

and determine whether T is an isomorphism.

8. (10pts) Let V and W be linear spaces. Let T be an isomorphism from V to W. Assume that $f_1, f_2, \cdots, f_n$ form a basis of V. Show that $T(f_1), T(f_2), \cdots, T(f_n)$ is a basis of W.