Problem 1  For the first part: the equation of the plane orthogonal to a given \( \vec{v} \in \mathbb{R}^3 \) is \( \vec{x} \cdot \vec{v} = 0 \). Applying this formula to our problem and writing \( \vec{x} = (x, y, z) \), we get
\[ x + 2y + 3z = 0. \]
For the second part, we have a formula for the reflection of \( \vec{w} \) in a plane \( P \):
\[ \text{ref}_P(\vec{w}) = \vec{w} - 2\vec{w}_\perp \]
where \( \vec{w}_\perp \) is the component of \( \vec{w} \) perpendicular to \( P \). But this is just
\[ \vec{w} - 2(\vec{w} \cdot \hat{v})\hat{v}, \]
where \( \hat{v} \) is a unit vector normal to \( P \). Applying this formula with \( \hat{v} = \frac{1}{\sqrt{13}}[123], \vec{w} = [456] \), we get
\[ \text{ref}_P(\vec{w}) = \frac{1}{7} \begin{bmatrix} -2 \\ -25 \\ -48 \end{bmatrix} \]

Problem 2  First we find the elementary row operations that reduce \( A \) to reduced row echelon form (if possible):
\[
\begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow 
\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow 
\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix}
\]
\[
\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

1
Since $A$ can be reduced to the identity matrix by elementary row operations, $A$ is invertible. Applying these same row operations to the $3 \times 3$ identity matrix gives us $A^{-1}$:

$$A^{-1} = \begin{bmatrix}
-1 & 1 & 1 \\
-2 & 1 & 2 \\
2 & -1 & -1
\end{bmatrix}$$

**Problem 3** Let $\vec{v} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Note that the plane $P$ defined by $x + z = 0$ is just the plane normal to $\vec{v}$. The problem demands a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ with kernel $\vec{v}$ and image $P$. Such a transformation $T$ is given by $T(\vec{x}) = \text{proj}_P \vec{x}$. To see this, note that $\vec{v}$, being normal to $P$, is in the kernel of $T$. Moreover, $T(\mathbb{R}^3) = P$ by definition. So this is the transformation we’re looking for. To find the matrix of $T$, write

$$\text{proj}_P \vec{x} = \vec{x} - \vec{x}_\perp = \vec{x} - (\vec{x} \cdot \vec{v})\vec{v}.$$ 

$$= (I - \text{proj}_v)\vec{x}.$$ 

The matrix of $\text{proj}_v$ is (from HW 3)

$$B = \begin{bmatrix}
1/2 & 0 & 1/2 \\
0 & 0 & 0 \\
1/2 & 0 & 1/2
\end{bmatrix}$$

Thus the matrix of $T$ is $I - B$, or

$$\begin{bmatrix}
1/2 & 0 & -1/2 \\
0 & 1 & 0 \\
-1/2 & 0 & 1/2
\end{bmatrix}$$

2