110.201 Linear Algebra
5th Quiz

April 22, 2005

Problem 1 Using determinant rules, find the determinant of the matrix

\[ A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 4 & 4 \\ 1 & 4 & 9 & 9 \\ 1 & 4 & 9 & 16 \end{bmatrix}. \]

Problem 2 True or false, with reason if true and counterexample if false:

1. If \( A \) and \( B \) are identical except in the upper-left corner, where \( b_{11} = 2a_{11} \), then \( \det B = 2\det A \).

2. The determinant of a matrix is the product of the pivots.

3. If \( A \) is invertible and \( B \) is singular, then \( A + B \) is invertible.

4. If \( A \) is invertible and \( B \) is singular, then \( AB \) is singular.

Problem 3 An invertible linear map \( L: \mathbb{R}^n \to \mathbb{R}^n \) is called orientation preserving if \( \det(A) > 0 \), and orientation reversing otherwise.

a) Let \( T_n(x) \) be the opposite of the identity map in \( \mathbb{R}^n \), i.e.

\[ T_n(x) = -x. \]

Is \( T_n \) orientation preserving or orientation reversing?

b) Prove that for any invertible map \( L: \mathbb{R}^n \to \mathbb{R}^n \), the map \( LL^t: \mathbb{R}^n \to \mathbb{R}^n \) is orientation preserving.

c) The linear map \( \text{ref}_V : \mathbb{R}^3 \to \mathbb{R}^3 \) takes a vector \( \bar{x} \) to its reflection \( \text{ref}_V(\bar{x}) \) in a two-dimensional subspace \( V \subset \mathbb{R}^3 \). Is \( \text{ref}_V \) orientation preserving or reversing? Does the answer depend on \( V \)?

[Hint: Try to find a basis in which the matrix for \( \text{ref}_V \) is simple].