Problem 1  Find the determinant of the \( n \times n \) matrix
\[
A = \begin{bmatrix}
0 & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & 1 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 1 & \cdots & 0 & 0 \\
1 & 0 & \cdots & 0 & 0
\end{bmatrix}
\]

Problem 2  Let \( A \) be an \( n \times n \) matrix obeying the equation
\[ A^2 = A. \]

a) What are the possible values of \( \det(A) \)? Why?

b) Let \( V \) be the image of \( A \) and \( m = \dim V \) Find all relationships between \( m, n \) and the values for \( \det(A) \) you found above.

[An acceptable statement would be something like “If \( \det(A) = \ldots \), then \( \ldots \).”]

Can \( m = n \)? If so, and \( m = n \), what can you say about \( A \)?

Problem 3  Suppose that two square matrices satisfy the following identity \( AB = -BA \). Find the flaw in the following argument, showing a counterexample:

Taking determinants gives \( \det(A)(\det B) = -(\det B)(\det A) \), so either \( A \) or \( B \) must have zero determinant. Thus \( AB = -BA \) is only possible if \( A \) or \( B \) is singular.