# Syllabus for Vector Calculus 110.202, Spring 2005

January 31, 2005

> Philosophy is written in this immense book that stands ever open before our eyes (I speak of the Universe), but it cannot be read if one does not first learn the language and recognize the characters in which it is written. It is written in mathematical language, and the characters are triangles, circles, and other geometrical figures, without which it is not humanly possible to understand a word; without these, philosophy is confused, wandering in a dark labyrinth.

[Galileo Galilei, *Il Saggiatore* (The Assayer), in *Opere*, V. 6 p. 197, tr. J. Barbour]

> Computers are useless, they can only give you answers ...

[Attributed to Pablo Picasso, probably apocryphal.]

Text: *Vector Calculus* by J. Marsden, A.Tromba, Fifth Edition, W.H. Freeman & Co. Prerequisite: $C-$ or better, or advanced placement credit, in Calculus II

## 1 Overview and course description

The techniques of vector calculus were codified in the early 20th century to provide a useful and sophisticated language for physics and mechanics. This is a math course, but some of most beautiful and compelling examples of applications come from the subject's roots in the 'hard' sciences, and we will hopefully encounter many examples.

**First steps: vector geometry.** This course begins with a condensed account of vector geometry and a little bit of linear algebra, and quickly moves to study derivatives of functions of several variables. A **vector** is basically an algebraic way of specifying a direction; the notion is important, for example, in the mechanics of objects moving in space, and once the basic notions (velocity, acceleration, ...) are in place we can verify, in the course of one lecture, that Kepler's laws follow from Newton's: which was in some sense Newton's life's work. (One can argue, however, that his life's work was really calculus itself.) This is the sort of *application* that we have in mind.

**Functions of several variables.** The next focus of attention in the course is optimization of functions of several variables: this generalizes the familiar one-variable maximum and minimum problems with the theory of *critical points* being now replaced by that of calculating tangents; however the problem in higher dimension is more complicated simply because there is more room for conflicts and constraints. The main technical tool here is the gradient $\nabla f$ of a function. As one example of an *application* of the study of optimization, we prove the existence of eigenvectors (whatever those are) for symmetric matrices (likewise).

**Vector fields.** By this point we will be used to concepts which are genuinely high-dimensional. An important central idea is a vector **field** $\mathbf{F}$, which assigns to each point of space a direction (examples: the wind, or the classical electric and magnetic fields, which were the original source of the notion). Classical continuum mechanics (concerned for example with heat, fluid flow, electromagnetic fields ...) is expressed in the language of vector calculus, which was invented for exactly this purpose; in particular, there is a sophisticated set of tools (called **divergence**, denoted $\nabla \cdot \mathbf{F}$ and **curl**, denoted $\nabla \times \mathbf{F}$) for dealing with *derivatives of vector fields*, and the latter third of the course will be devoted to learning how to use these tools.

The fundamental theorem of calculus is a central organizing principle for this course, but that may not become apparent till we're nearing its end. That theorem relates integrals to derivatives, but we won't spend much time on integration *per se*: the emphasis will be on practical computations, using the formula for change of variables, which emphasizes the utility of **coordinate systems** appropriate to the problem at hand. The one-dimensional

FTC (Fundamental Theorem of Calculus) calculates the integral over an interval of a derivative to the value of the function at the endpoints, and its higher analogues relate integrals of derivatives over surfaces or volumes to integrals over the boundaries of these regions. The main practical problem in evaluating these quantities involves finding a good description of relatively complicated domains of integration: these effective descriptions are called **parametrizations**; the formal definition occurs late in the text (§7.3) but they will be an important theme in the course from the beginning. These variations on the FTC (Gauss and Stokes' theorems) played an important role in the history of physics, relating (for example) electric flux across a surface to the charges contained inside. We will cover this topic at the end of the course.

## 2 Exams and grading

**Homework:** the **first two** assignments are due in section, the same week as they assigned. Starting with the **third homework** we will collect the homework on Mondays (the week after homework has been assigned), and you will receive it back either on Wednesday during lecture or at the end of the week in section. Occasionally, homework will be replaced by an in-class quiz during section, but that will be announced in advance. Some of the problems assigned as homework may be done in class, and others will be assigned. Late assignments will not be accepted.

Note: we will eventually deviate from the syllabus a little; if so, it will be your responsibility to keep track of the assignments.

**Exams.**

- first midterm: in class **Weds. 9 March**
- second midterm: in class, **Weds. 6 April**
- final exam: **May 12**, 9-12 noon, location tba.

Note: the final exam will be weighted toward the material at the end of the course. Mathematics is by its nature cumulative, so if you fall behind, it becomes nonlinearly difficult to recover.

**Grades:** following with departmental guidelines, we will assign at least one standard deviation (about a third) of B's, adjusted so as to maximize the number of A's.

If $M_1$ and $M_2$ are your two midterm test scores, $F$ is the final exam score, and $H$ is the homework grade [all on a basis of one hundred points] then your final score will be

$$\frac{1}{20}[3H + 4(M_1 + M_2) + 9F] \; ;$$

in other words, the homework is worth very nearly as much as an in-class test, so it's worth your while to take it seriously.

Note: there will be **no makeup exams**, basically because it seems to be in practice impossible to administer them fairly. In case of an excused absence, you will receive a weighted average of your score on the **subsequent** exams.

# 3 General issues

**Study groups:** especially encouraged when working on your homework or otherwise. Although exchange of ideas is encouraged, the work you turn in should be entirely your own.

**Help Room**: open from 9:00 to 5:00 on weekdays (as well as on some evenings- check the schedule) in Krieger 213. You are strongly encouraged to seek help from the graduate students on duty.

**Special needs:** students with special needs requiring extra time or special assistance during exams must inform the instructor and the Office of Academic Advising at the beginning of the course. This information will remain confidential.

**Academic Honesty.** Academic dishonesty is unacceptable. All cases of academic dishonesty are investigated thoroughly and punished severely.