Past Exam Problems in Integrals

Prof. Qiao Zhang
Course 110.202

November 15, 2004

The following is a list of the problems concerning integrals that appeared in the midterm and final exams of Calc III (110.202) within the last several years. You may use them to check your understanding of the relevant material. Some other exam problems may be found at

`http://reserves.library.jhu.edu/access/reserves/findit/exams/110/110202.php`

Note: These problems do not imply, in any sense, my taste or preference for our own exam. Some of the problems here may be more (or less) challenging than what will appear in our exam.

1. Evaluate the double integral

$$\iint_D x^2 y^2 \, \mathrm{d}x \, \mathrm{d}y$$

over the triangle D with vertices $(0,0), (1,0), (1,2)$.

2. The shape of a platform is given by

$$x^2 + y^2 \le (2-z)^2, \qquad 0 \le z \le 1.$$

   (a) Describe this shape in cylindrical coordinates.

   (b) Find the volume of this platform.

3. Find the volume of the solid enclosed by the two paraboloids $z = 2(x^2+y^2)$ and $z = 1 + x^2 + y^2$.

4. Find a parametrization for the surface defined by the intersection of the plane $x + y + z = 1$ with the cylinder $x^2 + y^2 = 1$. Use that parametrization to calculate the area of the surface.

5. Suppose that a particle follows the path $\mathbf{r}(t) = 2\cos(2t)\mathbf{i} + 2\sin(2t)\mathbf{j} + 3t\mathbf{k}$. Then find the total length of the path travelled by the particle from $t = 0$ to $t = \pi/4$.

6. Set up a double integral in polar coordinates to find the volume of the solid which is bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 2y$. DO NOT EVALUATE!

7. Evaluate the integral

$$\int_0^1 \int_y^1 \cos\left(\frac{1}{2}\pi x^2\right)\, \mathrm{d}x\, \mathrm{d}y.$$

8. Find the volume of the region under the surface $z = x^2 + 2y$ and over the region in the first quadrant between the line segment connecting $(0, 2)$ and $(2, 0)$ and the curve $y = 4 - x^2$.