MATH 202 — MIDTERM I

Department of Mathematics
Johns Hopkins University

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NAME: ____________________________ SIGNATURE: ___________________________

SECTION NUMBER:_____________

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1. This exam has 6 pages including this cover.
2. No books, notes or calculators are allowed.
3. The correct answer is worth **zero** points. For full credit we must be able to see how you got your answer.

PROBLEM	POINTS	SCORE
1	25	
2	25	
3	25	
4	25	
TOTAL	100	

1. a.[10 pts] Find the tangent plane to the surface

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid y = x^3 - yz\}$$

at the point $P(2, 1, 7)$.

b.[15 pts] Find the points on S where the tangent plane is parallel to the plane

$$9x - y - 3z = 4$$

2. Consider the path in $\mathbb{R}^3$ given by

$$\gamma(t) = (2t - 1, t + 2, t^2), \quad t \in \mathbb{R}$$

and let α the plane given by the equation

$$\alpha : \quad x + 2y + z = 1$$

a. [15 pts] Let $\phi(t) = \text{dist}^2(\gamma(t), \alpha)$ i.e. the square distance from $\gamma(t)$ to the plane α. Compute $\phi'(t)$ using the chain rule.

b. [10 pts] Determine t_0 for which $\gamma(t)$ is closest to α.

3. The three points $A(1,2,1), B(1,0,0)$ and $C(0,2,0)$ determine a plane ABC in $\mathbb{R}^3$.

a. [10 pts] Find a normal vector $\overrightarrow{N}$ to the plane ABC.

b. [15 pts] Let P the projection of the origin O onto the plane ABC. (i.e. P is the point in the plane ABC such that OP is perpendicular on the plane ABC). Determine the coordinates of P.

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ the map given by

$$f(x,y) = \begin{cases} \frac{x^2y+xy^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

a. [5pts] Compute $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$.

b. [5pts] Compute the directional derivative $\partial_{\overrightarrow{v}} f(0,0)$ along any unit vector

$$\overrightarrow{v} = \cos(\theta)\,\overrightarrow{i} + \sin(\theta)\,\overrightarrow{j}$$

c. [5pts] Prove that f is not differentiable at $(0,0)$.

d. [10pts] Determine the direction of greatest increase at $(0,0)$. Express the answer as a unit vector.

Helpful things:

I. The distance from a point $P = (x_0, y_0, z_0)$ to the plane α given by the equation $AX + BY + CZ + D = 0$ is $\mathrm{dist}(P, \alpha) = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2+B^2+C^2}}$.

II. Some trigonometric identities:

$$\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$$
$$\sin(\theta) + \cos(\theta) = \sqrt{2}\,\sin(\theta + \frac{\pi}{4})$$

III. Picture for Problem 3: