PROBLEM ON COMPUTING FLOWS

Consider the surface (paraboloid) Σ given by $x^2 + y^2 - z = 0$, $0 \leq z \leq 1$.

1) Parametrize Σ using cylindrical coordinates (that is, viewing it as the surface obtained by rotating $z = y^2$ around the z-axis). Compute the flow of $\mathbf{r}$ in two ways:
A) by going through the following steps:

- compute the unit normal vector $\overrightarrow{n}_\Sigma$ separately by normalizing the gradient of $g(x, y, z) = x^2 + y^2 - z$. Make sure your unit normal vector is pointing outwards.
- Compute $\mathbf{r} \cdot \overrightarrow{n}_\Sigma$
- Compute the area element dS.
- Integrate $\int\int_\Sigma (\mathbf{r} \cdot \overrightarrow{n}_\Sigma) dS$

B) by going through the following steps (assume one has parametrization $x = x(u, v)$, etc...):

- Use the determinant formula $\overrightarrow{F} \cdot dS = \begin{vmatrix} F_1 & F_2 & F_3 \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix} dudv$ (provided $T_u \times T_v$ point in the direction of $\overrightarrow{n}_\Sigma$).
- compute the flow as $\int\int_\Sigma \overrightarrow{F} \cdot dS = \int\int (\text{determinant})\, dudv$

2) Do the same thing as in 1), this time parametrizing Σ as a graph surface: $x = x$, $y = y$, $z = x^2 + y^2$ so the parameters are $(x, y) \in D$, the unit disk in $\mathbb{R}^2$.

3) Put a lid on Σ to enclose a solid region W. Use Gauss divergence theorem to compute the flow of $\mathbf{r}$ through Σ. Compare your result with 1) and 2).