PRACTICE PROBLEMS FOR MIDTERM 2

1. Integrals

1.1. Compute the volume of the sphere of radius R by computing the integral $\int\int_{D(O,R)} \sqrt{1-x^2-y^2}dxdy$ using polar coordinates.
Here $D(0,R)$ is the disk $x^2+y^2 \leq R^2$.

1.2. Compute the integral $\int\int_D \frac{dxdy}{\sqrt{x^2+y^2}}$ where D is the domain in $\mathbb{R}^2$ bounded enclosed by the parabola $y=x^2$ and the line $y=x$.
Hint: use polar coordinates to parametrize the domain D by $x = r\cos\theta, y = r\sin\theta$, with $0 \leq \theta \leq \frac{\pi}{4}$ and $0 \leq r \leq \frac{\sin\theta}{\cos^2\theta}$.

1.3. Compute $\int\int_D x^2y\,dxdy$ where D is the domain in $\mathbb{R}^2$ bounded by the lines $y=0$, $y=1-x$ and $y=x+1$.

1.4. Compute $\int\int_D xy^2\sqrt{x^2+y^2}\,dxdy$ where D is the region of the disk $x^2+y^2 \leq 1$ where $x \geq 0$ and $y \leq 0$.

1.5. [Ex. 10, p. 327] Find the volume bounded by the graph of $f(x,y) = 1+2x+3y$, the rectangle $[1,2] \times [0,1]$ and the four vertical planes bounding the rectangle.

1.6. $\int\int_{[-1,1]\times[0,1]}(x^2+y^2)dxdy$.

1.7. Determine the volume of the region in $\mathbb{R}^3$ bounded by the paraboloid $z = 4-x^2-y^2$ and the xy-plane.

1.8. Calculate the volume bounded above by the graph $z = 1-x^3-y^3$ and below and on the sides by the coordinate planes $x=0, y=0, z=0$.