# PRACTICE PROBLEM

Let $H$ the spiral staircase

$$x = u\cos\theta, y = u\sin\theta, z = \theta, \quad 0 \leq u \leq 1, 0 \leq \theta \leq 2\pi$$

(You might take a look at the picture on p. 464.)

a) Parametrize the boundary curve $\partial H$. The boundary has four parts

$$\partial H = \gamma_1 + \gamma_2 - \gamma_3 - \gamma_4$$

corresponding to

$$\theta = 0, \quad r = 1, \quad \theta = 2\pi, \quad u = 0$$

respectively. Pay attention to the orientation of the boundary. (Note that $H$ is parametrized by the box $[0,1] \times [0,2\pi]$ in the $(u,\theta)$-plane. The boundary of $H$ corresponds to the boundary of that box.)

2) Consider the vector field $\overrightarrow{F} = x\mathbf{i} - y\mathbf{j}$. Prove that $\overrightarrow{F} = \text{curl}\,\overrightarrow{G}$, with $\overrightarrow{G} = xy\mathbf{k}$. (why does such a vector field $\overrightarrow{G}$ exist?)

3) Compute the flow of $\overrightarrow{F}$ through $H$ by using 2) and Stokes theorem.

4) Find a scalar function (potential) $\phi(x,y,z)$ such that $\nabla\phi = \overrightarrow{F}$ (why does such a potential exist?)

5) Compute the circulation of $\overrightarrow{F}$ around the oriented boundary $\partial H$.