PRACTICE PROBLEMS FOR MIDTERM 2

1. Extrema

1.1. Find and classify the critical points of $f(x, y) = x^3y^2(12 - x - y)$.

1.2. Find the points on the ellipse

$$23x^2 + 14xy + 23y^2 = 17$$

closest to, and furthest from, the origin: i.e.: maximize the function x^2+y^2 subject to that constraint.

1.3. Find the absolute maxima and minima of the function $f(x, y) = 5x^2-2y^2+10$ on the disk $x^2 + y^2 \leq 1$.

1.4. Find the extrema of $f(x, y) = xy$ subject to the constraints $2x+3y \leq 100, 0 \leq x$, $0 \leq y$.

1.5. Find the extreme points of $f(x, y, z) = x + y + z$ subject to the constraints $x^2 + y^2 = 5$ and $y + 2z = 3$.

1.6. Find the points furthest from and closest to the origin on the curve $x^6+y^6 = 1$.

1.7. The temperature on the spherical surface $x^2 + y^2 + z^2 = 1$ is given by $T(x, y, z) = xy + yz$. Find all the hot spots.