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11.
$$\int\int\int_W f(x,y,z)dV = \int_0^1 \int_{-2}^2 \int_0^{y^2} (2x-y+z)dzdydx$$
$$= \int_0^1 \int_{-2}^2 ((2x-y)z + \frac{z^2}{2})|_0^{y^2} dydx = \int_0^1 \int_{-2}^2 ((2x-y)y^2 + \frac{y^4}{2})dydx = \frac{176}{15}$$

18.
$$\int\int\int_W f(x,y,z)dV = \int_{-1}^1 \int_{-2\sqrt{1-y^2}}^{2\sqrt{1-y^2}} \int_0^{x+2} zdzdxdy = \int_{-1}^1 \int_{-2\sqrt{1-y^2}}^{2\sqrt{1-y^2}} (\frac{x^2}{2}+2x+2)dxdy$$
$$= \int_{-1}^1 (\frac{8}{3}(1-y^2)^{\frac{3}{2}} + 8\sqrt{1-y^2})dy$$

Now if we use the change of variable : $y = \sin\theta$ and the identity : $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ we have :

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8}{3}\cos^4\theta + 8\cos^2\theta d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2}{3}(1+2\cos 2\theta + \cos^2 2\theta) + 4(1+\cos 2\theta)d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2}{3}(1+2\cos 2\theta + \frac{1}{2}(1+\cos 4\theta)) + 4(1+\cos 2\theta)d\theta = 5\pi$$

19. The projection of the intersection of the paraboloid $z = 4x^2+y^2$ and $y^2+z=2$ on the xy plane is $2x^2+y^2=1$.

$$V = \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{1-2x^2}}^{\sqrt{1-2x^2}} \int_{4x^2+y^2}^{2-y^2} dzdydx = \frac{\sqrt{2}}{2}\pi$$

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23.We use the cylindrical coordinates.

$$\int\int\int_W (x^2+y^2+2z^2)dV = \int_{-1}^2 \int_0^{2\pi} \int_0^2 (r^2+2z^2)rdrd\theta dz = 48\pi$$

27. We use the cylindrical coordinates.

$$V = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{10-2r^2}}^{\sqrt{10-2r^2}} rdzdrd\theta = 4\sqrt{2}\pi\frac{5\sqrt{5}-8}{3}$$

29. We use the cylindrical coordinates.

$$\int\int\int_W (2+x^2+y^2)dV = \int_0^{2\pi}\int_0^4\int_3^{\sqrt{25-r^2}} (2+r^2)rdzdrd\theta = \frac{656\pi}{5}$$

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11.

$$\int\int_D \delta(x,y)dxdy = \int_0^9\int_0^{\sqrt{x}} xydydx = \frac{243}{2}$$

$$\int\int_D x\delta(x,y)dxdy = \int_0^9\int_0^{\sqrt{x}} x^2ydydx = \frac{9^4}{8}$$

$$\int\int_D y\delta(x,y)dxdy = \int_0^9\int_0^{\sqrt{x}} xy^2dydx = \frac{2}{21}3^7$$

$$\overline{x} = \frac{\int\int_D x\delta(x,y)dxdy}{\int\int_D \delta(x,y)dxdy} = \frac{27}{4}$$

$$\overline{y} = \frac{\int\int_D y\delta(x,y)dxdy}{\int\int_D \delta(x,y)dxdy} = \frac{12}{7}$$

Online Home works :

1. The critical points of f are :

(a) The Points where f has no derivative :

$$\{(x,y)\in\mathbb{R}^2 : 3x^2+3y^2 \le 1 \ \ and \ \ x+y=0\}$$

The value of f on this set is 0 .

(b) The roots of $f_x = f_y = 0$.

$$f(x,y) = \begin{cases} \arcsin(x+y) & x+y>0 \\ -\arcsin(x+y) & x+y<0 \\ 0 & x+y=0 \end{cases}$$

so

$$f_x(x,y) = f_y(x,y) = \begin{cases} \frac{1}{\sqrt{1-(x+y)^2}} & x+y>0 \\ -\frac{1}{\sqrt{1-(x+y)^2}} & x+y<0 \\ DNE & x+y=0 \end{cases}$$

which have no zeros.

(c) Critical points on the boundary. We find them by the lagrange method. If we define $g(x,y) = x^2 + y^2 - \frac{1}{3}$ then we should solve find the solutions of the system :

$$\nabla f(x,y) = \lambda \nabla g(x,y) \qquad g(x,y) = 0$$

If $x + y > 0$ then :

$$\begin{cases} \frac{1}{\sqrt{1-(x+y)^2}} = \lambda(2x) \\ \frac{1}{\sqrt{1-(x+y)^2}} = \lambda(2y) \\ x^2 + y^2 = \frac{1}{3} \end{cases}$$

If $x + y < 0$ then :

$$\begin{cases} -\frac{1}{\sqrt{1-(x+y)^2}} = \lambda(2x) \\ -\frac{1}{\sqrt{1-(x+y)^2}} = \lambda(2y) \\ x^2 + y^2 = \frac{1}{3} \end{cases}$$

we get $(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}})$ from the first equations and $(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}})$ from the second system of equations. The value of f at these points is $\arcsin(\sqrt{\frac{2}{3}})$. So the maximum of f is $\arcsin(\sqrt{\frac{2}{3}})$ and its minimum is 0.

2. We use the cylindrical coordinates. But we use y axis instead of the z axis and so we have:

$$x = r\sin\theta \quad z = r\cos\theta$$

$$\int\int\int_T |x| dxdydz = 2\int\int\int_{T, x>0} xdxdydz = 2\int_0^{\pi}\int_0^2\int_0^{e^{-y}} r\sin\theta r dr dy d\theta$$

$$= \frac{4}{9}(1 - e^{-6})$$