]

1

Page 98-100

34. a)Define $F(x,y,z) = x^2 + xy - xz$. Then the level set of $F(x,y,z) = 2$ is the given surface.

(b) Solving for z in the equation $x^2 + xy - xz = 2$ we find $z = \frac{x^2+xy-2}{x}$

Page 114-116

14. The value of the $\frac{xy}{x^2+y^2}$ on the line $y = mx$ is $\frac{m}{1+m^2}$ when $x \neq 0$, which depends on m. So the limit doesn't exist.

17. If we rewrite $\frac{x^2-xy}{\sqrt{x}-\sqrt{y}} = \frac{x(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}{\sqrt{x}-\sqrt{y}} = x(\sqrt{x}+\sqrt{y}) = x^{3/2} + x\sqrt{y}$ then the limit is 0 using $Theorem 2.5$.

33. f had no limit at $(0,0)$. Because on the line $x = 0$ the value of f is 1, while on the line $y = 0$ its value is -1. So f is not continuous.

42. (a) $|x| = \sqrt{x^2} \leq \sqrt{x^2+y^2} = \|(x,y)\|$. The other inequality will be proved in the similar way.

(b) From a we have $|x| \leq \|(x,y)\|$ so $|x|^3 \leq \|(x,y)\|^3$ and similarly $|y|^3 \leq \|(x,y)\|^3$. So by the triangle inequality $|x^3+y^3| \leq |x^3| + |y^3| \leq 2\|(x,y)\|^3 = 2(x^2+y^2)^{\frac{3}{2}}$.

(c) From b we have $|x^3+y^3| \leq 2(x^2+y^2)^{\frac{3}{2}}$. dividing both sides of this inequality by x^2+y^2 we have $\frac{|x^3+y^3|}{x^2+y^2} \leq 2\sqrt{x^2+y^2} < 2\delta$ by assumption.

(d) Suppose $\epsilon > 0$ is given. If we choose any $\delta > 0$ such that $2\delta < \epsilon$ then if $\|(x,y)\| < \delta$ then by c $\frac{|x^3+y^3|}{x^2+y^2} \leq 2\sqrt{x^2+y^2} < 2\delta < \epsilon$. So $\lim \frac{|x^3+y^3|}{x^2+y^2}$ is zero as $(x,y) \to (0,0)$.