

9. f is positive for $0 \leq x \leq 1$, $-1 \leq y \leq 2$. so the volume is

$$\begin{aligned} \int_{x=0}^1 \int_{y=-1}^2 (2x^2 + y^4 \sin \pi x) dy dx &= \int_{x=0}^1 (2x^2 y + \frac{1}{5} y^5 \sin \pi x) \Big|_{-1}^2 dx \\ &= \int_{x=0}^1 (6x^2 + \frac{33}{5} \sin \pi x) dx = (2x^3 - \frac{33}{5\pi} \cos \pi x) \Big|_0^1 = 2 + \frac{66}{5\pi} \end{aligned}$$

14.

$$\begin{aligned} \int_{-2}^3 \int_0^1 |x| \sin \pi y dy dx &= \int_{-2}^3 |x| (-\frac{1}{\pi} \cos \pi y) \Big|_0^1 dx = \int_{-2}^3 \frac{2}{\pi} |x| dx \\ &= \frac{2}{\pi} (\int_{-2}^0 (-x) dx + \int_0^3 x dx) = \frac{13}{\pi} \end{aligned}$$

7.

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3y dy dx = \int_0^1 3y \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx = \int_0^1 6\sqrt{1-x^2} dx = \frac{3\pi}{2}$$

15. First we find the intersection of

$$y = x^2 + 2, y = 2x^2 - 2$$

$$x^2 + 2 = 2x^2 - 2 \Rightarrow x = \pm 2, y = 6$$

$$\begin{aligned} \iint_D (x - 2y) dA &= \int_{x=-2}^2 \int_{y=2x^2-2}^{x^2+2} (x - 2y) dy dx = \int_{x=-2}^2 (xy - y^2) \Big|_{2x^2-2}^{x^2+2} dx \\ &= \int_{x=-2}^2 (3x^4 - x^3 - 12x^2 + 4x + 4) dx = (\frac{3}{5}x^5 - \frac{1}{4}x^4 - 4x^3 + 2x^2 + 4x) \Big|_{-2}^2 = -\frac{128}{5} \end{aligned}$$

22. First we find the intersection of

$$y = 2x, y = 1 - 2x - x^2$$

in the first quadrant.

$$2x = 1 - 2x - x^2 \Rightarrow x = -2 + \sqrt{5}$$

because x must be positive. Now the area is

$$\begin{aligned} \int_0^{-2+\sqrt{5}} \int_{2x}^{1-2x-x^2} dy dx &= \int_0^{-2+\sqrt{5}} (y)|_{2x}^{1-2x-x^2} dx = \int_0^{-2+\sqrt{5}} (1-4x-x^2) dx \\ &= (x-2x^2-\frac{1}{3}x^3)|_0^{-2+\sqrt{5}} = \frac{10\sqrt{5}-22}{3} \end{aligned}$$

26. (a) The Volume of the graph of $z = x^2 - y^2 + 5$ can be computed by evaluating this integral :

$$\iint_D (x^2 - y^2 + 5) dA = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2 - y^2 + 5) dy dx$$

note that on the region $x^2 + y^2 \leq 4$ the function $z = x^2 - y^2 + 5$ is positive.

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1. (a)

$$\begin{aligned} \int_0^2 \int_{x^2}^{2x} (2x+1) dy dx &= \int_0^2 (2x+1)y|_{x^2}^{2x} dx = \int_0^2 (2x+1)(2x-x^2) dx \\ &= \int_0^2 (-2x^3 + 3x^2 + 2x) = (-\frac{1}{2}x^4 + x^3 + x^2)|_0^2 = 4 \end{aligned}$$

(c) First we find the intersection of $y = x^2, y = 2x$:

$$x^2 = 2x \Rightarrow x = 0, 2.$$

so they intersect at $P = (0, 0)$ and $Q = (2, 4)$. also note that

$$y = x^2 (x \geq 0) \Rightarrow x = \sqrt{y}$$

$$y = 2x \Rightarrow x = \frac{y}{2}$$

so we can rewrite the integral in a as:

$$\int_0^4 \int_{\frac{y}{2}}^{\sqrt{y}} (2x+1) dx dy = \int_0^4 (x^2+x)|_{\frac{y}{2}}^{\sqrt{y}} dy = \int_0^4 (\frac{y}{2} + \sqrt{y} - \frac{y^2}{4}) dy = (\frac{1}{4}y^2 + \frac{2}{3}y^{\frac{3}{2}} - \frac{1}{12}y^3)|_0^4 = 4$$

12. If we change the order of integration we have :

$$\int_0^\pi \int_y^\pi \frac{\sin x}{x} dx dy = \int_0^\pi \int_0^x \frac{\sin x}{x} dy dx = \int_0^\pi (\frac{\sin x}{x})y|_0^x dx = \int_0^\pi \sin x dx = 2$$