Problem Set 10

From "Dummit & Foote"

1. nn. 4, 7, 9 p. 278

2. n. 5, 6 p. 283

3. n. 7 p. 293; n. 1 p. 306

Further exercises

4. Prove or find a counterexample to the following statement:

a quotient of a P.I.D. by a prime ideal is again a P.I.D.

5. Prove that if R is a P.I.D. and D is a multiplicative closed subset of R, then $D^{-1}R$ is also a P.I.D.

6. Let $R = \mathbb{Z}[\sqrt{-6}]$. Prove that 2 and $\sqrt{-6}$ are irreducible in R. Show that R is not U.F.D. and give an explicit ideal of R which is not principal.

7. In the Gaussian integers find $G.C.D.(11 + 7i, 18 - i)$.

8. Let $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$ be the prime field with p elements. Let $f(x) \in \mathbb{F}_p[x]$ be a polynomial of degree $n \geq 1$. Prove that $\mathbb{F}_p[x]/(f(x))$ has p^n elements.

9. Determine all the prime ideals in the ring $\mathbb{Z}[x]/(2, x^3 + 1)$.

10. Determine in $\mathbb{F}_2[x]$ the $G.C.D.(x^3 - 1, x + 1)$.