Problem Set 2

From "Dummit & Foote"

1. nn. 4, 5 p. 35; n. 3 p. 36

2. nn. 2, 7, 11, 14, 17, 25 p. 40-41

3. nn. 6,7,8,10 p. 49

4. nn. 6,10 p. 53

Further exercises

5. 1. Show that any group G of order 2,3,5 is cyclic (i.e. $\exists g \in G$ s.t. $G = \{g^n \mid n \in \mathbb{Z}\}$).

6. Let G and G' be two groups, $g \in G$ and $g' \in G'$. Show that $|(g, g')| = l.c.m.(|g|, |g'|)$, where $(g, g') \in G \times G'$. Assume now that the 2 groups are cyclic, let $|G| = n < \infty$ and $|G'| = m < \infty$. Determine a necessary and sufficient condition on the orders so that $G \times G'$ is cyclic.

7. Is the map:

$$\psi : \mathbb{Z}/6\mathbb{Z} \to \mathbb{Z}/8\mathbb{Z}, \qquad \psi(\overline{a}) = [a]$$

a homomorphism of groups? If not, define a homomorphism between these 2 groups. How many homomorphisms can one define between these 2 groups? Explain.