<u>Problem Set 3</u>

The following exercises are taken from "Dummit & Foote"

1. nn. 10, 11, 13, 15, 18, 26 p. 60–61

2. nn. 3, 4, 5, 9, 14 p. 85-86

Further exercises

3. Assume that a group G acts transitively on a finite set A and let H be a normal subgroup of G. Let $\mathcal{O}_1, \dots \mathcal{O}_r$ be the distinct orbits of H in A.
   (1) Prove that G permutes the sets $\mathcal{O}_1, \dots \mathcal{O}_r$ in the sense that for each $g \in G$ and for each $i \in \{1, \dots r\}$, there is an integer $j \in \{1, \dots r\}$ such that $g\mathcal{O}_i = \mathcal{O}_j$, where $g\mathcal{O} = \{ga \mid a \in \mathcal{O}\}$. Prove that G is transitive on $\{\mathcal{O}_1, \dots \mathcal{O}_r\}$. Deduce that all orbits of H in A have the same cardinality.
   (2) Prove that if $a \in \mathcal{O}_i$, then $|\mathcal{O}_i| = |H : H \cap G_a|$, where G_a is the stabilizer of $a \in G$.

4. Show that if $G/Z(G)$ is cyclic ($Z(G) =$ center of the group G), then G is commutative.

5. Let G be a group and $H < G$. Determine a bijection between the set of left-cosets and that of right-cosets of H in G.