

## Problem Set 4

The following exercises are common to both editions (2nd and 3rd) of “Dummit & Foote”

1. n. 6, 8, 11, 18 p. 96-97.
2. nn. 3, 8 p. 102-103.
3. n. 9 p. 113
4. n. 10 p. 119

Further exercises

5. Show that if a group  $G$  has two normal, proper subgroups  $H, K$  of index  $p > 1$ ,  $p$  prime number, s.t.  $H \cap K = \{1\}$ , then:

$$|G| = p^2 \text{ and } G \text{ is not cyclic.}$$

6. Let  $g_1, \dots, g_r$  be representatives of the conjugacy classes of the finite group  $G$  and assume these elements pairwise commute. Prove that  $G$  is abelian.

7. Let  $G$  be a finite group,  $|G| = n$  and let  $H$  be a subgroup of  $G$ ,  $|H| = m < n$ . Call  $H = H_1, \dots, H_s$  the conjugate subgroups of  $H$ . Show that  $\cup_{i=1}^s H_i \subsetneq G$ .  
(Hint: note that  $s = |G : N_G(H)|$  and  $H \leq N_G(H)$ )