Problem Set 4

The following exercises are taken from “Dummit & Foote”

1. nn. 4,6,8,10,12 p. 95–96

2. nn. 8,9 p. 101

Further exercises

3. Show that if a group G has two normal, proper subgroups $H,\ K$ of index $p > 1$, p prime number, s.t. $H \cap K = \{1\}$, then:

$$|G| = p^2 \text{ and } G \text{ is not cyclic.}$$

4. Let $g_1, \dots g_r$ be representatives of the conjugacy classes of the finite group G and assume these elements pairwise commute. Prove that G is abelian.

5. Let G be a finite group, $|G| = n$ and let H be a subgroup of G, $|H| = m < n$. Call $H = H_1, \dots H_s$ the conjugate subgroups of H. Show that $\cup_{i=1}^{s} H_i \subsetneq G$.
(Hint: note that $s = |G : N_G(H)|$ and $H \leq N_G(H)$)